RESEARCH ARTICLE

Divisibility Algorithm For Number 12

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ABSTRACT: Divisibility, which can mean dividing an integer by another integer without a remainder, also includes finding the remainder in division. In teaching mathematics; odd, even, prime, etc. states of numbers are important for integers and related subjects, it plays a role in determining many of a number's properties. The divisibility rules of certain numbers are known to students. However, the rules of some numbers are determined by the factors of that number. For example, in the divisibility rule by 12 states, "If the number is divisible by both 3 and 4, it is also divisible by 12." Our aim in this paper is to express the divisibility rule by 12 differently and independently from its multipliers (3 and 4) and to create a completely new algorithm suitable in order to achieve our goal. For our purpose, firstly, it was desired to create a systematic order with numbers that are a whole multiple of 12. Then, a new divisibility algorithm was developed by working with division including remainders. It has been proven that the algorithm works flawlessly in studies using multiple different sized numbers and their resulting mathematical explanation and proofs.

KEYWORDS: Divisibility, digits, remainders, twelve, division with & without remainders.

Introduction

"Divisibility" refers to dividing an integer by an integer without a remainder.[1] Divisibility is a subject that contains the information necessary to estimate the remainder in division with remainders, as equally as it is about division without remainders. In the teaching of this subject, the rules are given to the students in the beginning of their studies. Teaching continues by being reinforced with examples. However, in order to improve students' mathematical skills and provide more permanent learning, students can be given numbers that are divisible by an integer without a remainder, and they can be asked to examine whether there is a certain rule between them.[1,2] Thus, the logic of divisibility memorized as a set of rules can be grasped. Yet this would not stay as permanent knowledge without clear explanations rather than straightforward rules, which our paper won't be doing.

When studies on divisibility are examined, there are not many studies on the direct divisibility rules. Based on foreign sources, studies on divisibility rules with prime numbers lead the way.[3,4] The reason for this is that divisibility by a non-prime number is explained by divisibility to all prime factors of the input.[5]

Even though the majority of divisibility rules are prime factor-based, there are some well-known direct rules as well. Certain numbers (such as 2, 3, 4, 5, 7, 8, 9, 10, 11, and 13) have their own divisibility rules that are used by everyone.

For example, A number:

- To be divisible by 2 without a remainder, it must be even.
- To be divisible by 3 without a remainder, the sum of the digits of the number must be a multiple of 3.
- To be divisible by 4 without a remainder, the number in the last two digits of the number must be "00" or a number divisible by 4 without a remainder.

Of course, all these divisibility rules have a proof, a premise. However, even though students know these rules by heart, most of them have no idea why and how these apply.[1] Different activities can be developed to increase students' mathematical reasoning skills of ministry topics and to enable them to produce solution algorithms. In this research, we have examined the number 12, whose rule is accepted such as: "Numbers that are divisible by both 3 and 4 are divisible by 12," and is among the numbers that don't have their own divisibility rules.

Purpose:

During the preparation of the project, our first goal was to answer the question: "Can we find a divisibility rule for any number **other than** the numbers whose rule is known and used by everyone?". In line with the answers we gave to the questions asked, we aim to fill a gap in this field and to the number "12". Just as it was recommended in its teaching, it was started by considering the numbers that are multiples of 12 and their common features.

The study was planned and implemented in order to determine the numbers divisible by 12 and to create an algorithm for finding the remainder and if it is divisible or not.

Method

In this project, an algorithm was created to calculate the divisibility of a number by 12 and what the remainder of dividing the number by 12 is.

This divisibility algorithm, which was created on the basis of existing divisibility rules, includes situations such as four operations, steps, and number values. While creating the algorithm, in the first place, a study was carried out on numbers that are a whole multiple of 12 accommodating the "1k-2k" structure of 12, and with the data obtained in "division by 12 without a remainder", it was aimed to reach the conclusion of "division by 12 with a remainder". Continuing on, the answer to the question: "what will be the remainder as a result of dividing a number by 12" was attempted to be achieved. An

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algorithm has been developed by taking into account the place values of the numbers whose divisibility is investigated through numerous trials. While applying the algorithm, two similar ways are followed depending on whether the number is odd or even.

If the number is "even":

a. The number in the one's digit of the number is separated from the numbers in the other digits.

E.g; A three-digit ABC number is separated from the C number on ones-digit from the three-digit ABC number to obtain the two-digit number AB and the number C. (AB and C)

b. Then, half of the separated number in the ones digit (C/2) is subtracted from the remaining digits (AB two-digit number) (AB - C/2).

c. The number obtained after these operations is divided by 12.

d. If there is a remainder, it is "multiplied by 2 and subtracted from the smallest multiple of 12 bigger than the multiplication result".

Thus, the remainder of the division of the first number by 12 is calculated. If the remainder is "0", the number is divisible by 12 (without a remainder).

If the number is "odd":

a. The number in the one's digit of the number is separated from the numbers in the other digits.

E.g; A three-digit ABC number is separated from the C number on ones-digit from the three-digit ABC number to obtain the two-digit number AB and the number C. (AB and C)

b. Then, "half of 1 more than 1 [(C+1)/2]" of the number in the ones digit is subtracted from the remaining digits (AB two-digit number) [AB-(C+1)/2].

c. The number obtained after these operations is divided by 12.

d. If there is a remainder, it is "multiplied by 2, subtracted from the smallest multiple of 12 bigger than the multiplication result, and subtracted by 1", respectively.

Thus, the remainder of the division of the first number by 12 is calculated.

These steps can be repeated one after the other according to the number of digits of the number we want to calculate its divisibility by 12. It is important to consider the "odd"ness and "even"ness of the remaining numbers after each repetition. A much clearer explanation of the prediscussed topics with examples can be found in sections: 3.1, 3.2, 3.3, 5 and appendices.

Findings:

First of all, we can see that the algorithm works without errors by experimenting with smaller numbers of 3-4 digits. Below are examples where the algorithm is applied only once. Then, while working with larger numbers with multiple steps, the steps of the application are explained according to the changing even and odd situations.

Application of the algorithm on an even number once:

First of all, we can see that the algorithm works without errors byexperimenting with smaller numbers of 3-4 digits. Below are examples where the algorithm is applied only once. Then, while working with larger numbers with multiple steps, the steps of the application are explained according to the changing even and odd situations.

Application of the algorithm on an even number once:

I. The number in the ones digit of the number is separated from the other digits.

→ For the number 3146, the number is divided into two as 314 and 6

II. Half of the number left is subtracted from the remaining number

$314 - \frac{6}{2}$

III. The result obtained as a result of subtraction is divided by 12. (314–3=311)

$$\begin{array}{r|l} 311 & 12 \\ \underline{-24} & 25 \\ 71 & \\ \underline{-60} & \\ 11 & \end{array}$$

IV. The remainder obtained as a result of division is multiplied by 2.

11 x 2 = 22

V. The result of the multiplication is subtracted from the smallest positive integer of 12 bigger than the result of the multiplication itself.

24 - 22 = **2**

Result of the Operation: The remainder of the division of 3146 by 12 is **2**.

P.S: IV. If the number found as a result of the operation is a multiple of 12, we can say that the number is divisible by 12 without doing V. Operation.

Application of the algorithm on an odd number once:

I. The number in the ones digit of the number is separated from the other digits. For the number 1035, the number is divided into two as 103 and 5.

II. Half of 1 more than the number left is subtracted from the separated digit.

$$103 - \frac{(5+1)}{2}$$

III. The result obtained as a result of the subtraction is divided by 12. (103–3=100)

$$\begin{array}{r|l} 100 & 12 \\ \underline{-96} & 8 \\ 4 & \end{array}$$

IV. The remainder obtained as a result of the division is multiplied by 2. 4x2=8

V. The result of the multiplication is subtracted from the smallest positive integer of 12 bigger than the result of the multiplication itself.

12-8=4

VI. In the second operation, "+1" is subtracted from the number obtained as a result of the previous step.

4 - 1=3

VII. Result of Operation: The remainder of the division of 1035 by **12 is 3**.

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Application of the algorithm on any number multiple times:

I. The number in the ones digit of the number is separated from the other digits. For the number 689814, the number is divided into two as 68981 and 4.

II. Half of the number left is subtracted from the remaining number. (If the reserved number is odd; half of 1 more than the number is subtracted.)

Here the newly formed number is odd. In the next step, half of the number will be subtracted. In this way, single or double cases will be taken into account.

$$68981 - \frac{4}{2} = 68979$$

Operations 1 and 2 are repeated sequentially until a 2-digit number is obtained. (Let's call this operation a step reduction operation.) (We put a "-" sign in order to remind the steps with odd numbers in order not to make mistakes.)

Step 1 $\quad 689814 \rightarrow 68981 \quad 4$

$$68981 - \frac{4}{2} = 68979$$

Step 2 (-) $\quad 68979 \rightarrow 6897 \quad 9$

$$6897 - \frac{(9+1)}{2} = 6892$$

Step 3 $\quad 6892 \rightarrow 689 \quad 2$

$$689 - \frac{2}{2} = 688$$

Step 4 $\quad 688 \rightarrow 68 \quad 8$

$$68 - \frac{8}{2} = 64$$

As a result of repeated operations, the remainder is found from the division of the 2-digit number by 12.

$$\begin{array}{r|l} 64 & 12 \\ \underline{-\,60} & 5 \\ 4 \rightarrow c & \end{array}$$

To make the operations more understandable, we designated the remainder we found as the number c.

IV. Multiply number c by 2.

12 – 4 = 8

V. Subtract the result of the IV.th operation from the smallest multiple of 12 bigger than 2*c.

12 - 8 = 4

The 4th and 5th operations are repeated in reverse (step 4-3-2-1) as the number of steps in the step reduction process.

For the number in step 4 → 4 x 2 = 8

12 - 8 = 4

For the number in step 3 → 4 x 2 = 8

12 - 8 = 4

For the number in step 2(-) → 4 x 2 = 8

12 - 8 (-1) = 3

(Here 1 is subtracted because the number in Step 2 is odd.)

For the number in step 1 → 3 x 2 = 6

12 - 6 = **6**

The number found after these operations gives the remainder of the first number (689814) divided by 12.

Result of Operation: The remainder of the division of 689814 by 12 is **6**.

As you can see, techniques used for different situations are given in the algorithm in the "Method" section. Based on whether the number is odd or even, the divisibility of numbers by 12 and their remaining states are revealed as a result of simple four operations. You can examine Appendix-1 for an example of a number that is exactly divisible by 12, Appendix-2 for the example where the algorithm is applied more than once and all numbers are odd while applying, and Appendix-3 for the example where the algorithm is applied more than once, and all numbers are even while applying.

Mathematical basis of the algorithm:

For clarity, let's take a 3-digit number only. Let this number be ABC. If C is even:

C is separated from the ABC number. If we write this mathematically; ABC= 100A+10B+C= 10.(AB)+C

The number obtained by subtracting C and dividing by 10 will be AB. Then half of C is subtracted from this number AB.

$$\frac{ABC - C}{10} - \frac{C}{2}$$

If we write the equivalent of this expression:

$$\frac{ABC - C}{10} - \frac{C}{2} = \frac{ABC - C}{10} - \frac{5C}{10} = \frac{ABC - C - 5C}{10} = \frac{ABC - 6C}{10}$$

number is obtained. Let's call this last number x. According to the algorithm, this number x was divided by 12 and the remainder was found. Now let's substitute 12k+m for the number x. The goal is to find m, the remainder. This number m would be multiplied by 2 and subtracted from 12 to find the remainder from the division of ABC by 12 (just like in the 4th and 5th steps of the algorithm). Therefore, the remainder should be 12 - 2m.

$$x = \frac{ABC-6C}{10} = 12k + m$$

Let x be this equation. Since the main purpose is the divisibility of ABC by 12, let's write the equation of ABC.

With the product of insides and outsides method:

$$ABC = \mathbf{120k} + \underline{10m} + \mathbf{6C}$$

is found. Let's examine this number now. It is clear that 120k is a multiple of 12 here. Since the initially accepted number C is even, it can be seen that C itself contains a factor of 2 and 6C is a multiple of 12. The only imprecise term here is 10m. To ensure this, let's write 10m=12m-2m and rewrite the equation:

$$ABC = 120k + 6C + 12m\ \mathbf{-2m}$$

The bolded part in the new equation is the remainder of ABC divided by 12. However, since the remainder cannot be negative, 12 must be added until it becomes positive. So for m<12, the remainder of ABC dividing by 12 is 12-2m. This shows that the algorithm is working correctly. Thus, the conditions for the divisibility of even numbers by 12 are proved.

If C is odd:

According to the algorithm, half of 1 more of C is taken and the operations are repeated. Let C+1=D. As the number is odd (ABC), the divisibility of the even number ABD by 12 is checked. Thus, if the remainder from the division of the ABD by 12 is **y**, it is obvious that the remainder of the division of the ABC by 12 will be **y-1**.

Conclusion

An algorithm has been created in order to perform a divisibility rule by 12 in a different way other than dividing by known and prime factors. Many examples have been studied to show and control how this algorithm we have created works in odd or even numbers. In order to increase clarity, examples of different situations are included in our study in the upcoming sections. In order to find the mathematical basis of our algorithm, which gives error-free results in all cases, a proof is presented with the help of the number values, variables, and division-divisibility information.

As a result, since the algorithm developed allows us to obtain smaller numbers by decrementing the digits in very extensive numbers, it has introduced a brand-new method and a distinct rule for divisibility by 12.

Appendices:

Appendix-1

I. The number in the ones digit of the number is separated from the other digits.

For the number 288, the number is divided into two as 28 and 8.

II.Half of the number left is subtracted from the remaining number.

$$28 - \frac{8}{2} = 24$$

III. The result obtained as a result of subtraction is divided by 12. (28-4=24)

$$\begin{array}{r|l} 24 & 12 \\ \underline{-\,24} & 2 \\ 00 & \end{array}$$

Results

● If the remainder in the division operation is "0" as a result of the 3 sequential operations; It means the number is divisible by 12.

● Thus, we can say that the number 288 is divisible by 12 (without a remainder).

Appendix-2

The algorithm is applied more than once, and all numbers are even while applying:

I. The number in the ones digit of the number is separated from the other digits.

For the number 378784, the number is divided into two as 37878 and 4.

II. Half of the number left is subtracted from the remaining number.

Operations 1 and 2 are repeated sequentially until a 2-digit number is obtained.

i. Step

$$378784 \rightarrow 37878 \quad 4$$

$$37878 - \frac{4}{2} = 37876$$

ii. Step

$$37876 \rightarrow 3787 \quad 6$$

$$3787 - \frac{6}{2} = 3784$$

iii. Step

$$3784 \rightarrow 378 \quad 4$$

$$378 - \frac{4}{2} = 376$$

iv. Step

$$376 \rightarrow 37 \quad 6$$

$$37 - \frac{6}{2} = 34$$

III. As a result of repeated operations, the remainder is found from the division by 12.

$$\begin{array}{r|l} 34 & 12 \\ \underline{-\,24} & 2 \\ 10 \rightarrow a & \end{array}$$

To make the operations more understandable, we called the remainder we found as the number "a".

IV. Multiply a with 2.

10x2=20

V. Subtract the result of the IVth step from the smallest multiple of 12 bigger than a*2.

24-20 = 4

The 4th and 5th operations are repeated as many times as the number of steps in the IInd step.

iv. → 10x2=20
24-20= 4

iii. → 4x2 = 8
12-8 = 4

ii. → 4x2 = 8
12-8 = 4

i. → 4x2 = 8
12-8 = **4**

● The number found after these operations gives the remainder of the first number discussed by dividing it by 12.

● The remainder of the division of 378784 by 12 is **4**.

Appendix-3

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The algorithm is applied more than once, and all numbers are odd while applying:

I. The number in the ones digit of the number is separated from the other digits.

For the number 267673, the number is divided into two as 26767 and 3.

II. Half of 1 more than the number left is subtracted from the remaining number.

Operations 1 and 2 are repeated sequentially until a 2-digit number is obtained.

I. Step
$267673 \rightarrow 26767 \quad 3$
$26767 - \frac{(3+1)}{2} = 26765$

ii. Step
$26765 \rightarrow 2676 \quad 5$
$2676 - \frac{(5+1)}{2} = 2673$

iii. Step
$2673 \rightarrow 267 \quad 3$
$267 - \frac{(3+1)}{2} = 265$

iv. Step
$265 \rightarrow 26 \quad 5$
$26 - \frac{(5+1)}{2} = 23$

III. As a result of repeated operations, the remainder is found from the division by 12.

$$\begin{array}{r|l} 23 & 12 \\ -\,12 & 1 \\ \hline 11 \rightarrow b & \end{array}$$

To make the operations more understandable, we called the remainder we found as the **number b.**

IV. Multiply b with 2.

11x2 = 22

V. Subtract the result of the IVth step from the smallest multiple of 12 bigger than b*2 and subtract by 1.

24-22 = 2,

2 - 1 = 1

The 4th and 5th operations are repeated as many times as the number of steps in the IIrd step.

iv. → 11x2 = 22
24-22 = 2
2-1 = 1

iii. → 1x2 = 2
12-2 = 10
10-1 = 9

ii. → 9x2 = 18
24-18 = 6
6-1 = 5

i. → 5x2 = 10
12-10 = 2
2-1 = 1

- The number found after these operations gives the remainder of the first number discussed divided by 12.
- The number found after these operations gives the remainder of the first number discussed divided by 12.
- The remainder of the division of 267673 by

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