

Correlations Among Fundamental Equations in High School Physics Coursebook of Cambridge University Press

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ABSTRACT: Mathematics is an application tool to grasp built-in physics basics. Fundamental physics knowledge is inevitable in our routine life and will improve our scientific vision. Mathematics and Physics are interrelated concepts during high school learning and herculean tasks for almost all high school students. The process of teaching physics concepts to students is two-fold. One is to establish mathematical rigor for physics concepts. The other is the practical demonstration of these concepts. The present work aims at providing a pathway to learning fundamental physics efficiently. We have assorted the equations in high school physics coursebooks (AS and A2 parts) published by Cambridge University Press according to their inherent relationships. The interlinkage of these equations will enhance the ability of students to acquire basic physical concepts. To our knowledge, such an interlinkage has not been reported in previous literature. The present article also intends to achieve the core aim of physics learning, which includes an analysis of basic equations to describe real-time applications. Hence, the relations among the equations are the central theme of discussions to assist high school students in learning physics courses efficiently.

KEYWORDS: High School Students; Physics Coursebook; Fundamental Equations; Correlations; Pathway for Learning Physics.

■ Introduction

Physics represents complexity compared to other courses for high school students because of its connection to mathematics. Knowledge of physics is fundamental for various disciplines of science and technology. For instance, the underlying basics behind magnetic resonance imaging (MRI) during health check-ups would perplex both the patient and the physician. A student, who is comfortable with sophisticated physics concepts, will competently realize the processes of nature after mastering this course. Teaching physics concepts to students is like a double-edged sword. An ardent teacher teaching the concepts constructively to students will excel. Alternatively, students failing to cope with advanced physics concepts will undergo problems during their academic careers after high school. In the context of providing a possible remedy to the tricky situation, the current work analyzes the relations among basic equations in the high school physics coursebook. The intention is to propose a feasible pathway for high school students to understand various equations in the high school coursebook efficiently.

Cambridge Assessment International Education (CAIE) is part of Cambridge University Press & Assessment – an organization that provides world-leading academic research, learning, and assessment globally, backed by the first-class teaching and research departments of Cambridge University. The AS (first-year course) and A2 (second-year course) Levels coursebooks published by CIE have served as the exclusive coursebooks for more than 530,000 students from over 130 countries each year. The present authors have found that these coursebooks introduce the fundamental equations individually, which are classified into different branches regarding various physical phenomena and principles. However, many equations

are related to each other. So far, there is no report on critical evaluation and correlation for fundamental equations in both course books and syllabi of the Cambridge International AS & A2 Level Physics 9702.¹⁻⁴ This is also the case for the physics coursebooks currently used by high school students in China, five the USA, six and India.⁷ In addition, to the best of our knowledge, most of the articles published from 2019 to now in the International Journal of High School Research (IJHSR) focus on scientific investigations, eight and only two publications^{9,10} concentrate on learning issues. For high school students, both learning and scientific research are equally important.

To efficiently teach or learn physics in high school, we combine the contents of the Physics 9702 coursebooks to establish relationships among basic equations from different chapters. After describing the methods used in the present work, we divide our critical analyses of these fundamental physical equations into two parts. Section 1 consists of a deep analysis of the fundamental equations of AS coursebook in the first year. In contrast, section 2 presents the correlations between the first-year basic equations and the second-year counterpart of the A2 coursebook. The established correlations demonstrate our views on efficiently teaching or learning core concepts in physics. Following our proposed pathway, it is expected that high school students will grasp school physics more efficiently.

■ Methods

In the present work, a strategy to interpret physics principles to describe the behavior of various phenomena of our interest is using the system's parameters, sketches, and mathematical formulae. A mathematical model is an essential representation of a real problem and has built-in assumptions and approxima-

tions. The vital issue is deciding whether the model is sound and whether the results are interpreted physically. If a model is not reasonable, its shortcomings are analyzed to provide an improved formulation for the model. In a nutshell, the mathematical formulation of a physical problem could be a cyclic process. Figure 1 illustrates a flowchart with the principles of mathematical modeling, which has the following steps:

Step 1: identify the physical situation; Step 2: convert the physical condition into a mathematical model by introducing parameters/variables and using various known physical laws; Step 3: find the solution to the mathematical problem; Step 4: interpret the result in terms of fundamental physical laws and compare it with observations or experiments; Step 5: accept the model if the result is in good agreement with observations or experiments. Otherwise, modify the hypotheses/assumptions according to the physical situation and go back to Step 2.

This is the ordinary procedure that physics research follows. During the whole process, establishing physical models is the core issue that thwarts many students from mastering the knowledge during their high school physics learning. In addition, the analysis of physical phenomena under natural conditions will throw students into a dilemma. To solve these problems, we attempt to construct relations among different equations in different chapters to efficiently propose a pathway for learning high school physics.

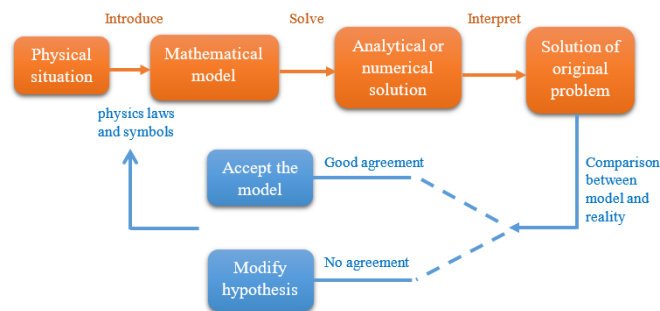


Figure 1: Schematic to interpret physics principles using system parameters, sketches, and mathematical formulae. This flow chart expresses the ordinary procedure for physics research. The core issue during the procedure is the establishment of physically sound models confusing many high school students.

■ Results and Discussion

Evaluation and correlation of fundamental equations in the AS-level physics course book:

The AS level section introduces basic rules of physics, including mechanics, electronic circuits, waves, and atomic structure. Table 1 summarizes the fundamental equations at this level. Electronic circuits in chapters 8-11 and atomic structure in chapter 15 are independent of the other chapters that are about mechanical physics. The detailed theories about atoms (quantum mechanics) are difficult for high school students to visualize. Consequently, simple rules about atoms are introduced in chapter 15, and a rational choice is to set this chapter at the end of AS level during the first-year education. It is inevitable for students to understand the basics beforehand from chapters 1-14 to achieve in-depth knowledge about atomic structure. Learning basics (i.e., energy, power, electron, and so on) during the first year is for the sake of

smoothly understanding advanced physics concepts during the second year.

Electromagnetism, quantum mechanics, optics, and other topics in the AS level physics coursebook have underlying mathematical equations: e.g., magnetic flux, photoelectric effect, Bragg's law, etc., that constitute the base for higher physics education. Chapters 1 to 7 are about kinetics which elaborates on some basic rules of the motions, followed by chapters 8 to 15, describing basic knowledge about electronic circuits and waves.

The crux of AS level education is understanding physical observations and expressing them in mathematical equations via standard or independent parameters. There are two cores in AS level education: force analysis and electricity analysis. The subsequent discussions below are about the two cores.

Table 1: Fundamental equations for AS level chapters (Chapters 1 to 15). For the meanings of variables and symbols in these equations, please refer to Table 3.

Chapter	Equation	Chapter	Equation
Chapter 1: Kinematics	$v = d / t$ (1)	Chapter 8: Electric current	$\Delta Q = I \times \Delta t$ (21) $I = nAve$ (22) $R = V / I$ (23) $\Delta W = V \times \Delta Q$ (24) $P = VI$ (25)
Chapter 2: Accelerated motion	$a = \Delta v / \Delta t$ (2) $v = u + at$ (3) $s = ut + \frac{1}{2}at^2$ (4) $v^2 = u^2 + 2as$ (5)	Chapter 9: Kirchhoff's laws	$R = R_1 + R_2 + \dots$ (26) $R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots}$ (27)
Chapter 3: Dynamics	$F = ma$ (6) $F = mg$ (7)		
Chapter 4: Forces	$M = F \times x$ (8)		
Chapter 5: Work, energy and power	$W = F \times s$ (9) $E_p = mg \Delta h$ (10) $E_k = \frac{1}{2}mv^2$ (11) $P = F \times v$ (12)	Chapter 10: Resistance and resistivity	$R = \rho l / A$ (28)
		Chapter 11: Practical circuits	$V = E - IR$ (29)
		Chapter 12: Waves	$I = P / A$ (30)
Chapter 6: Momentum	$p_m = m \times v$ (13) $F = \Delta p / \Delta t$ (14)	Chapter 13: Superposition of waves	$\lambda = ax / D$ (31) $d \sin \theta = n \lambda$ (32)
Chapter 7: Matter and materials	$\rho = m / V$ (15) $P_s = F / A$ (16) $\Delta P = \rho gh$ (17) $k = F / x$ (18) $\varepsilon = x / L$ (19) $\sigma = F / A$ (20)	Chapter 14: Stationary waves	$v = f \lambda$ (33)
		Chapter 15: Atomic structure	

Firstly, innovation to understand fundamental equations effectively depends on several factors. For instance, the transformation of basic equations by multiplying parameters on both sides of an equation leads to new equations with a different perspective to study the system's properties. To show such new equations in numerical order after fundamental equations in AS and A2 levels, the fundamental equations from 16 to 29 chapters in the A2 level coursebook are listed in Table 2. As indicated in Tables 1 and 2, there are a total of 51 basic equations in AS and A2 levels. For example, we take Equation (4) in Table 1 to derive one new equation. Multiply-

ing "ma" to both sides of this equation leads to Equation (52).

$$ma \times s = ma \times ut + \frac{1}{2}ma^2t^2 \quad (52)$$

Table 3 lists the meanings of all the symbols used in the present article. The left side of this equation now represents work (force-displacement) when the force is applied, and the right side of the equation transforms to

$$ma \times s = W = mvu - mu^2 + \frac{1}{2}mv^2 - \frac{1}{2}mu^2 - mvu = \frac{1}{2}mv^2 - \frac{3}{2}mu^2 \quad (53)$$

The right side of Equation (53) was obtained by replacing all the parameters t in Equation (52) based on Equation (3). In this way, the relation between work and velocity helps to gain insight into the system's kinetic energy. Similarly, Equations (53) and (13) uncover the relationship between the system's momentum and work. The corresponding equation is as follows:

$$W = at \times \left(mu + \frac{1}{2}mat \right) = v_t \times \left(mu + \frac{1}{2}mv_t \right) \quad (54)$$

where v_t represents the increment of the velocity. The first and second terms in the bracket of Equation (54) are initial momentum and half of the increment of momentum, respectively.

The transformation of the fundamental equations at AS level will not only help students understand the basic knowledge but also give them a good command of the physical meanings behind these equations and associated parameters, which is the primary objective of AS level education.

Evaluation and correlation of fundamental equations in A2 level physics coursebook:

The second core discussion concerns the underlying linkage among different concepts from various chapters. Synchronously, the following discussions illustrate using a specific model for other physics concepts. Figure 2 shows the relations among different equations in the A2 level physics coursebook. The motion of objects is a basic theory that is also valid for describing the motion of celestials and charged particles. Except for thermal motion, there exists a wide range of applications for motion equations.

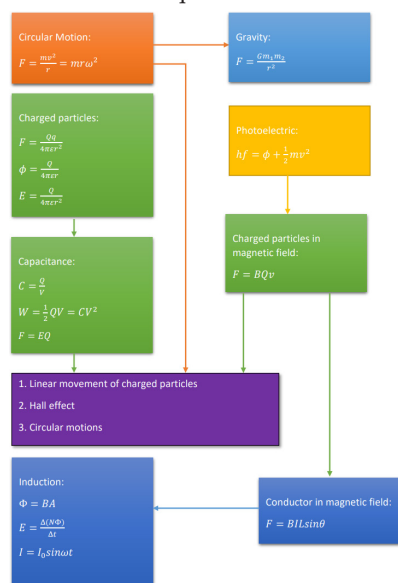


Figure 2: Fundamental equations and their correlations for A2 level chapters (Chapters 16 to 29). For the meanings of variables and symbols in these equations, please refer to Table 3. The core item in the A2 section is the movement of charged particles under electrostatic and magnetic fields. The trajectory of charged particles is described by circular motion equations combined with the forces under the electrostatic and magnetic fields. The photoelectric effect process can provide the charged particles.

The principle of circular motion describes celestial activities under the influence of gravity. Hence, combining Equations (34) and (35) yields the velocities of celestials. Similarly, Equation (41), in conjunction with Equation (34), can describe the electron's movement under a magnetic field. It is worth mentioning that the electrons moving in a magnetic field are associated with capacitance, coulomb law, induction, and the Hall effect, as shown in Figure 2.

In a capacitor system, the simplified model with two plates carrying opposite charges separated by a distance is appropriate to describe its capacitance in physics education. Applying an external magnetic field perpendicular to a charged particle's direction of motion will enhance the model's complexity in tracking the trajectory of the charged particles. Supposing that one electron with charge "q" moves from left to right in the middle of the capacitor, and the magnetic field "B" is applied perpendicular to its direction of motion, as shown in Figure 3, one obtains the force balancing electromagnetic equation:

$$qE = Bqv \quad (55)$$

Equation (55) calculates the initial velocity of the electron as per the capacitor model. Using Equations (1) and (4), one can calculate the position at which the electron hits the capacitance plate.

$$ma = qE - Bqv \quad (56)$$

Extending the analysis, the expression for Hall voltage is as follows:

$$V = Ed = \frac{Bld}{nAe} \quad (57)$$

Equation (57) originates from Equations (22) and (55). The intrinsic physical theory of the Hall effect is based on the electron convergence at one side of capacitance when the magnetic field is applied. The knowledge from chapters 21-25 describes the particle's movement in an external magnetic field, and the circular motion analysis is performed in chapter 16. Students realizing the coherency of the textbook would sharpen their physics skills.

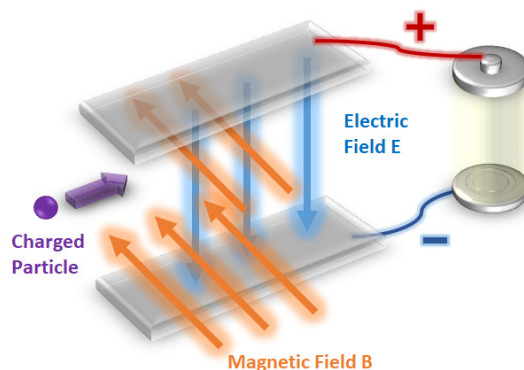


Figure 3: Charged particles moving under a magnetic field (orange arrow) and the electric field (indigo blue arrow) provided by capacitance plates. The equations of charged particle motion in these two fields will describe the trajectory of the particles, including several physics phenomena of circular motion, capacitance, electrostatic force, and the photoelectric effect.

Chapters 28 and 29, comprising the photoelectric effect and nuclear reactions, are an alternate perspective to realize the energy's nature. Unlike mechanics (e.g., kinetic energy), energy packets describe the photoelectric effect. For instance, assuming that energy is continuous as per mechanics, the electron emission should be continuous, but electrons eject from the materials after they receive appropriate energy packets (threshold energy). Equation (55) is still valid to describe the motion of an electron in a magnetic field as per the above discussions.

The generation of these charged particles can also be introduced into this model. Equation (50) explains the photoelectric effect. Similarly, the incorporation of the generation of charged particles from a nuclear reaction in chapter 29 is also feasible in the present analysis. Thus, the charged particles model is significant in high school physics education. Understanding the charged particle movement equations will facilitate understanding knowledge in these chapters.

The analysis of the motion of a charged particle in terms of equation 55 under an external magnetic field forms the fundamental of an atomic phenomenon. A charge (electrostatics) in motion produces a current that simultaneously induces a magnetic field (electrodynamics). An electron with a negative charge behaves as an intrinsic magnet moving under the influence of an atom's nuclear potential. The essence of electricity in conductors is the migration of electrons to nullify the potential (force) difference. In this line of thought, induction is a cause-effect phenomenon that occurs during electron motion in a conducting wire subjected to a varying magnetic field and vice versa.

Interestingly, we combine Equation (22) in Table 1 and Equation (42) in Table 2 to obtain the equations in the induction part. Defining q as the total charges in the wire moving in the external magnetic field and ωt as the angle between the wire and magnetic fields, one can obtain the electric current:

$$I = nAve = vq = \frac{F}{B \sin \omega t} \quad (58)$$

The electric current in chapter 8, the force in an external magnetic field in chapter 24, the motion of charged particles in a magnetic field in chapter 25, and the induction phenomenon in chapter 26 are the main focuses of the current analysis. Consequently, the relation between the wire moving in a magnetic field and the converse process current-carrying wire in a magnetic field has been constructed.

Table 2: Fundamental equations for A2 level chapters (Chapters 16 to 29). For the meanings of variables and symbols in these equations, please refer to Table 3.

Chapter	Equation	Chapter	Equation
Chapter 16: Circular motion	$F = \frac{mv^2}{r} = mr\omega^2$	Chapter 26: Electromagnetic induction	$\Phi = BA \quad (45)$
Chapter 17: Gravitational fields	$F = \frac{Gm_1m_2}{r^2} \quad (35)$	Chapter 27: Alternating currents	$E = \frac{\Delta(N\Phi)}{\Delta t} \quad (46)$
Chapter 18:			$I = I_0 \sin \omega t \quad (47)$

Oscillations			$P = VI = I^2 R = \frac{V^2}{R} \quad (48)$
Chapter 19: Thermal physics	$PV = n_m R T \quad (36)$	Chapter 28: Quantum physics	$E = hf = \frac{hc}{\lambda} \quad (49)$
Chapter 20: Ideal gases	$E_k = \frac{3}{2} KT \quad (37)$		$hf = \phi + \frac{1}{2} mv^2 \quad (50)$
Chapter 21: Uniform electric fields,	$F = \frac{Qq}{4\pi\epsilon r^2} \quad (38)$	Chapter 29: Nuclear physics	$\Delta E = \Delta mc^2 \quad (51)$
Chapter 22: Coulomb's law	$\phi = \frac{Q}{4\pi\epsilon r} \quad (39)$		
Chapter 23: Capacitance	$E = \frac{Q}{4\pi\epsilon r^2} \quad (40)$		
Chapter 24: Magnetic fields and electromagnetism	$F = BQv \quad (41)$		
Chapter 25: Motion of charged particles	$F = BIL \sin \theta \quad (42)$		
	$C = \frac{Q}{V} \quad (43)$		
	$W = \frac{1}{2} QV = CV^2 \quad (44)$		

Table 3: The variables and symbols for all the equations of Cambridge International AS and A2 level physics coursebooks.

Symbol	Meaning	SI-unit
v, u	speed	$m \cdot s$
d	distance	m
s	displacement	m
t	time	s
F	force	N
m	mass	kg
a	acceleration	m/s^2
M	momentum	$kg \cdot m/s$
W	work	J
Ep	gravitational potential energy	J
g	gravitational potential	$9.8 m/s^2$
Δh	change in height	m
E_k	kinetic energy	J
P	power or momentum	W
ρ	density	kg/m^3
V	volume	m^3
p	pressure	Pa
A	area	m^2
ϵ	strain	
σ	stress	Pa
$\Delta Q, Q, q$	charge	C
I	electric current	A
n	number density	m^{-3}
R	electric resistance	Ω
U	potential difference	V
E	emf of the source	V
λ	wavelength of the light	m
D	distance between the slits and the screen	m
a	separation between the center of the slits	m
x	separation between the center of the adjacent bright fringes	m
θ	angle	$^\circ$
f	frequency	Hz
n_m	mole fraction	
R_g	molar gas constant	$J/(mol \cdot K)$
K	Boltzmann constant	J/K
r	radius of circular motion	m
B	magnetic flux density	T

Φ	magnetic flux	$T \cdot m^2$
c	light speed	$3 \times 10^8 \text{ m/s}$

■ Conclusion

Firstly, this work carefully evaluates the fundamental equations in Cambridge International AS and A2 level physics coursebooks. Subsequently, we have established the correlations among these equations in these coursebooks for the first time. It is strongly suggested that high school students practice the transformation of these equations and perform the extension of these equations in different models. The presently proposed pathway will assist students in learning physics efficiently. In addition, following this pathway, students will gain the core cognition of physics, the optimal model construction, and their application in actual systems by combining different models.

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