■ RESEARCH ARTICLE

Time Evolution of the Mass of Primordial Black Holes in a Cosmic Microwave Background with a Time-Dependent Temperature

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ABSTRACT: Primordial black holes have been long theorized to exist, with their formation predicted to have been a short time after the Big Bang. In this paper, we discuss the different properties of primordial black holes, such as the possibility of primordial black holes being the seeds for supermassive black holes that are present in the center of galaxies and primordial black holes being a candidate for dark matter. Then, we show how we derived an equation for the evolution of the mass of a primordial black hole in a cosmic microwave background with a time-dependent temperature. Also, we discuss techniques that we used to solve the equation we derived and how we calculated the mass range of a primordial black hole in the early universe. The results of these calculations show the possible masses of primordial black holes that could account for all of the dark matter in the universe. If primordial black holes could explain the dark matter problem, then it would help extend our understanding of the composition of the universe and the formation of the early universe.

KEYWORDS: Physics and Astronomy; Astronomy and Cosmology; Black Holes; Primordial Black Holes; Thermodynamics.

Introduction

The existence of primordial black holes has been a topic of interest for scientists since 1967 when Yakov Borisovich Zel'dovich and Igor Dmitriyevich Novikov first suggested the existence of these black holes and also when Stephen Hawking studied these black holes in 1971.[1] These primordial black holes are predicted to have formed near the beginning of the universe soon after the Big Bang. The formation of primordial black holes cannot be due to stellar collapse like the black holes known to exist today because primordial black holes would have formed long before the existence of the first stars. Instead, it is theorized to be a result of irregularities in the uniformity of the cosmic microwave background. If primordial black holes do, in fact, exist, their existence means that the universe was not distributed equally in terms of mass, as there must have been some areas in the early universe with a high enough density to cause gravitational collapse and the formation of a black hole. The fact that primordial black holes could not have formed from stellar collapse also means that the initial mass of a primordial black hole is not bound by the same limits of initial mass that a black hole formed from stellar collapse is. This makes primordial black holes a strong topic of research among scientists, as it would better explain our understanding of the early universe and the distribution of mass in the early universe; it may explain the formation of supermassive black holes and the properties of primordial black holes also make it a possible candidate for dark matter.

In this paper, I will be exploring the possible existence of primordial black holes, how they could have survived to the present day, and the possibility of primordial black holes being the dark matter that is theorized to exist in our universe. The methods section of the paper will be a basic review of black holes and the properties and formation of a black hole. I will also discuss black hole thermodynamics, Hawking radiation, and black holes as blackbodies. There will then be a review of the cosmic microwave background and its temperature over time. This will explain the temperature of the cosmic microwave background in the early universe. The final part of the methods section will be an overview of primordial black holes, supermassive black holes, the possible formation of supermassive black holes, and primordial black holes as a candidate for dark matter. In the results and discussion section, I will explain the methods used to find the balance between the evaporation rate and absorption rate of primordial black holes and how the initial mass range of primordial black holes can be found using those rates. The methods to find this initial mass range involve creating an equation that finds the mass of a black hole as a function of time by taking into account the temperature of the cosmic microwave background over time, the absorption rate of a black hole from the cosmic microwave background, and the evaporation rate of a black hole due to Hawking radiation. I will then cover how I went about numerically solving the differential equation for the mass of the black hole as a function of time. This section will also explain why the equation is hard to solve and where the use of approximations was needed. It will also discuss the results on the initial mass range of primordial black holes found using the program. Finally, the conclusion section will summarize our findings and then compare our solution to other research that has been done on primordial black holes. Then, I will discuss the similarities and differences between our results and other research findings and compare our methods.

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Methods

Review of Black Holes:

According to Einstein's theory of General Relativity, a black hole is a region in spacetime where the force of gravity is too strong to allow anything to escape, including all matter and electromagnetic radiation. This is why black holes are completely invisible, as there is no light being reflected. The only way to detect black holes is by observing their gravitational effects on their surroundings. At the center of a black hole, there is a point where spacetime becomes infinitely curved, called a singularity. As the singularity is a singular point in spacetime, the volume of the singularity is zero, which means that the singularity also is a point of infinite density. Any object that falls into a black hole will fall towards this singularity without the possibility of escape. Black holes also contain an event horizon, which determines the boundary of a black hole. Once an object crosses the event horizon, it will not be able to escape and will inevitably fall toward the singularity. This also applies to any information from an event that occurs within a black hole, so any information that happens within the event horizon cannot be communicated to anyone outside of the black hole. The radius of a black hole, known as the Schwarzchild radius, can be determined as the distance from the singularity to the event horizon, which can be found with the equation

$$r = \frac{2GM}{c^2}, \tag{1}$$

with r being the radius of the black hole, G being the gravitational constant, M being the mass of a black hole, and c being the speed of light.

The only way astrophysical black holes can form is through stellar collapse. This occurs at the end of a star's lifetime when the star's mass gets drawn inward towards the center of gravity. While in stars not large enough to create a black hole, the force of gravity is not strong enough to overcome degeneracy pressure resulting in the formation of a white dwarf or a neutron star. However, in stars large enough to create a black hole, in which their remnants are larger than the Tolman–Oppenheimer–Volkoff limit, the force of gravity overwhelms the degeneracy pressure, resulting in the formation of a black hole.

Black Hole Thermodynamics:

The mass of a black hole over time depends on the thermodynamics of a black hole. The way in which black holes gain mass is through gaining energy from the cosmic microwave background, which converts to mass with Albert Einstein's famous $E = Mc^2$ equation. As stated in the previous section, the absorption rate of a black hole can be found using the Stefan-Boltzmann law, but this law is also effective in determining the emission rate of a black hole. This is because the Stefan-Boltzmann law determines the amount of energy radiated from a black body using its temperature. Black holes are the perfect black body as they absorb all radiation that comes into contact with them. Using the Stefan-Boltzmann law, the equation for the absorption rate of a black hole can be given as

$$P_{\text{Absorption}} = 4\pi r^2 \sigma T_{\text{CMB}}{}^4, \tag{2}$$

where $P_{Absorption}$ is the power absorbed by a black hole or the total amount of energy absorbed by a black hole in one second, r is the radius of the black hole, σ is the Stefan-Boltzmann constant, and T_{CMB} is the temperature of the cosmic microwave background.

Black holes emit radiation through Hawking radiation, which Stephen Hawking first theorized in 1974.[2] Hawking radiation happens when particles created from quantum fluctuations form near the event horizon of a black hole. While the particle and its antiparticle usually annihilate each other out of existence, when the particle pair forms near the event horizon of a black hole, sometimes one particle falls into the black hole. In contrast, the other is emitted out into the universe. However, the particle that falls into the black hole contains negative energy, which means the black hole will lose energy and mass from that particle, causing the black hole to evaporate slightly. To determine the amount of energy lost by a black hole due to Hawking radiation, the equation for the Stefan-Boltzmann law can be used again with the temperature of the black hole itself being used in the equation. The equation for the temperature of a black hole can be derived using the equation for the first law of thermodynamics,

$$dE = dQ - dW, \tag{3}$$

with dE being the change in energy of the black hole, dQ being the change in the heat of the black hole, and dW being the rate of work being done by the black hole. Since a black hole does no work on its surroundings, dW can be assumed to be zero. This equation is combined with the equation for the second law of thermodynamics,

$$dS = \frac{dQ}{T_{\text{BH}}}, \tag{4}$$

with dS being the change in entropy of the black hole and T_{BH} being the temperature of a black hole, to form the equation

$$T_{\text{BH}} = \frac{dE}{dS}. \tag{5}$$

Then, Einstein's $E = Mc^2$ equation can be differentiated to get

$$dE = dMc^2, \tag{6}$$

which can be inserted into the temperature equation found above to get

$$T_{\text{BH}} = \frac{dM}{dS}c^2 = \frac{dS}{dM}^{-1} c^2. \tag{7}$$

dS/dM can then be found by using the Bekenstein-Hawking formula for the entropy of a black hole,

$$S = \frac{k_{\text{B}}c^3}{4\hbar G}A, \tag{8}$$

and the formula for the area of the black hole,

$$A = \frac{16\pi G^2 M^2}{c^4}, \tag{9}$$

to find the entropy of a black hole in terms of mass,

$$S = \frac{4\pi k_{\text{B}} G M^2}{\hbar c}, \tag{10}$$

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which can be differentiated to

$$\frac{dS}{dM} = \frac{8\pi k_B G M}{\hbar c}. \quad (11)$$

When this equation for dS/dM is inserted back into the equation found for the temperature of a black hole, the result is the equation

$$T_{\mathrm{BH}} = \frac{\hbar c^3}{8\pi G M k_B}. \quad (12)$$

Using the Stefan-Boltzmann law, the equation for the emission rate of a black hole can be given as

$$P_{\mathrm{Emission}} = 4\pi r^2 \sigma T_{\mathrm{BH}}{}^4, \quad (13)$$

and T_{BH} can be inserted into the equation resulting in the equation

$$P_{\mathrm{Emission}} = 4\pi r^2 \sigma \left(\frac{\hbar c^3}{8\pi G M k_B}\right)^4. \quad (14)$$

Overview of the Cosmic Microwave Background and its Temperature Over Time:

The cosmic microwave background is the radiation left over from the Big Bang and its temperature can be used to determine the universe's temperature on a large scale. This is important because the cosmic microwave background's temperature determines a black hole's absorption rate with the Stefan-Boltzmann law. While the temperature of the cosmic microwave background is currently around 2.73 Kelvin, it has been decreasing ever since the Big Bang. It will eventually approach absolute zero at some point in the far future.[3] The reason for this decrease in the temperature of the cosmic microwave background is that the universe's expansion increases the wavelengths of radiation in the universe, causing a reduction in the radiation's energy, meaning the radiation's temperature is cooling down. Using the Friedmann equations, the temperature of the cosmic microwave background has decreased at different rates during different epochs. During the radiation-dominated era, the cosmic microwave background temperature was proportional to $t^{-1/2}$. In contrast, during the matter-dominated era, the temperature of the cosmic microwave background was proportional to $t^{-2/3}$.[4] Currently, the universe is in the dark energy-dominated era where the temperature of the cosmic microwave background is proportional to e-(Hubble constant x t).[5] Using these rates, the temperature of the cosmic microwave background can be determined at different points in time, which then can be used to determine the absorption rate of a black hole given its mass and point of time in the universe.

Primordial Black Hole Formation:

Primordial black holes could have been formed in the early universe during the radiation-dominated era, where the radiation density was higher than the density of matter in the universe. While the radiation-dominated era lasted until the universe was around 47,000 years old, the formation of primordial black holes is thought to have happened less than a second after the Big Bang.[6] This means that the formation of primordial black holes happened before the formation of the first stars in the universe, showing that the formation of primordial black holes could not have been due to stellar collapse. Therefore, primordial black holes would have formed from irregularities in the density of the early universe that were at a high enough density to cause gravitational collapse. Since primordial black holes could not have been formed by stellar collapse, the lower bound of the initial mass range of primordial black holes is not restricted by the Tolman-Oppenheimer-Volkoff limit. Black holes formed from stellar collapse have a lower bound on initial mass with the Tolman-Oppenheimer-Volkoff limit of around 1.5 to 3.0 solar masses.[7] Any star remnant with a mass below that limit will collapse into a neutron star or a white dwarf rather than a black hole. However, primordial black holes can form with an initial mass below that limit, meaning that if a black hole was found with a mass far below the Tolman-Oppenheimer-Volkoff limit, there is a strong possibility of the black hole being a primordial black hole, since there has not been enough time since the early universe for a black hole formed from stellar collapse to evaporate to that small of a mass from Hawking radiation. Even with this expanded mass range, there is still a lower bound on the initial mass of a primordial black hole that has survived to the present day, around 10^{11} kg. This is because any primordial black hole with an initial mass lower than 10^{11} kg would have completely evaporated before the present day due to Hawking radiation.

Supermassive Black Holes:

Supermassive black holes are black holes that are defined to be between 0.1 million and 10 billion solar masses.[8] Most large galaxies in our universe are believed to contain supermassive black holes at their centers, but their origin is unknown. One possibility is that primordial black holes could be seeds for supermassive black holes. The possibility of supermassive black holes forming from primordial black holes is more likely than the possibility of supermassive black holes forming from black holes that were formed from stellar collapse and grew from accretion. This is because primordial black holes would have had more time to grow by accretion, as they are predicted to have existed long before the existence of the first stars.[9]

Primordial Black Holes as Dark Matter:

The possibility of primordial black holes being a candidate for dark matter has been explored in many different studies, even though there has not been any evidence showing that this is the case. This is due to the properties of primordial black holes that show that it is entirely plausible that primordial black holes account for some, if not all, of the dark matter in the universe. One of the main properties that make primordial black holes a good candidate for dark matter is that primordial black holes are non-baryonic matter. Baryonic matter is matter made up of baryons, particles that are made up of an odd number of quarks, like protons and neutrons, making all atoms baryonic matter. Because primordial black holes formed during the radiation-dominated era, before Big Bang nucleosynthesis formed the baryonic matter in the universe, primordial black holes must be considered non-baryonic matter.[10] Most of the universe's dark matter is considered non-baryonic because it has not been directly observed and barely interacts with baryonic matter, making primordial black holes an entirely plausible candidate for dark matter. Primordial black holes are that would explain observations in galaxy halos that suggest the presence of dark matter.[11] However, many candidates for massive compact halo objects are composed of baryonic matter, making primordial black holes an interesting candidate as they are non-baryonic. The possibility of primordial black holes being all of dark matter was explored in a 2019 study, which concluded that there is still a possibility of primordial black holes in the mass range of 3.5x10^{-17} solar masses to 4x10^{12} solar masses or 7.0x10^{13} kg to 8x10^{18} kg making up all of the dark matter in the universe.[12] Because of this possibility, this is the mass range we will be using for the current mass of primordial black holes in my calculations of finding the mass of primordial black holes as a function of time.

■ Results and Discussion

Evolution Equation of the Mass of a Black Hole:

To find the evolution equation for the mass of a primordial black hole, the energy that the primordial black hole gains and loses over time must be found as that energy is directly converted to mass using Einstein's $E = Mc^2$ equation, which can be converted to

$$M = \frac{E}{c^2}. \tag{15}$$

Since the evolution equation is finding the mass of a primordial black hole as a function of time, the equation can be differentiated in terms of time to

$$\frac{dM}{dt} = \frac{\frac{dE}{dt}}{c^2}. \tag{16}$$

Power *(P)* is the derivative of energy with respect to time, so P can be substituted to *dE/dt* making the equation able to be written as

$$\frac{dM}{dt} = \frac{P}{c^2}. \tag{17}$$

The power of a black hole is equal to the power that the black hole is absorbing minus the power that the black hole is emitting, which can be written as

$$P = P_{\text{Absorption}} - P_{\text{Emission}}, \tag{18}$$

and using the equation for $P_{Absorption}$ and the equation for $P_{Emission}$, the power equation can be written as

$$P = 4\pi r^2 \sigma T_{\text{CMB}}{}^4 - 4\pi r^2 \sigma \left(\frac{\hbar c^3}{8\pi G M k_{\text{B}}}\right)^4. \tag{19}$$

P can then be inserted back into the equation, finding the change in mass of a black hole as a function of time, resulting in the equation

$$\frac{dM}{dt} = \frac{4\pi r^2 \sigma T_{\text{CMB}}{}^4 - 4\pi r^2 \sigma (\frac{\hbar c^3}{8\pi G M k_{\text{B}}})^4}{c^2}, \tag{20}$$

or

$$\frac{dM}{dt} = \frac{16\pi G^2 \sigma}{c^2} M^2 T_{\text{CMB}}{}^4 - \frac{\frac{\sigma c^{10} \hbar^4}{256\pi^3 G^2 k_{\text{B}}{}^4}}{M^2}. \tag{21}$$

Solving the Evolution Equation:

The equation found in the previous section must be integrated to find the mass of a primordial black hole when it formed in the early universe. To make the integration of this equation easier, the constants found in the equation can be combined to

$$A = \frac{16\pi G^2 \sigma}{c^2} \approx 1.74 \times 10^{-74}, \tag{22}$$

and

$$B = \frac{\sigma c^{10} \hbar^4}{256\pi^3 G^2 k_{\text{B}}{}^4} \approx 3.92 \times 10^{18}, \tag{23}$$

to make the equation

$$\frac{dM}{dt} = A M^2 T_{\text{CMB}}{}^4 - \frac{B}{M^2}. \tag{24}$$

A variable in the equation is T_{CMB}, which decreases over time. However, since a function to determine T_{CMB} over time is difficult to incorporate into the equation, we turned to splitting up the time axis into five different eras and treating T_{CMB}as a constant within these different eras, with era 1 being the period where the age of the universe was 10-12 seconds to 10-6 seconds, era 2 being the period from 10-6 seconds to 3 minutes, era 3 being the period from 3 minutes to 300,000 years, era 4 being the period from 300,000 years to 10^9 years, and era 5 being the 10^9 years to present day, which is estimated to be around 10^{10} years. Since the temperature of the cosmic microwave background is decreasing at a logarithmic rate, it can be assumed that the temperature during an era will mostly be a number near the temperature at the most recent point in an era. Therefore, the temperature of the cosmic microwave background at the most recent point in these eras can be used as the temperature for the entire era, which is given as 10^{13} K for era 1, 10^{10} K for era 2, 3000 K for era 3,10 K for era 4, and 3 K for era 5.[13] The integration of this equation is still difficult, which is why the program Mathematica was used to integrate this

$$\left[\frac{2arctan\left(\frac{A^{\frac{1}{4}} M T_{\text{CMB}}}{B^{\frac{1}{4}}}\right) + log\left(\frac{1-\frac{A^{\frac{1}{4}} M T_{\text{CMB}}}{B^{\frac{1}{4}}}}{1+\frac{A^{\frac{1}{4}} M T_{\text{CMB}}}{B^{\frac{1}{4}}}}\right)}{4 A^{\frac{3}{4}} B^{\frac{1}{4}} T_{\text{CMB}}{}^3}\right]_{M_{\text{i}}}^{M_{\text{f}}} = \Delta t, \tag{25}$$

equation. The integrated equation ended up becoming

where M_f is the mass of the primordial black hole at the present day, M_i is the mass of the primordial black hole at a given point in time, and Δt is the time between the present day and the given point in time.

With the present-day mass range of primordial black holes already known from the methods section, the mass range during each era can be calculated using the integrated equation. However, programs such as Mathematica and MATLAB could not provide results due to the equation's complexity, so we devised a way to solve the equation by splitting the equation

$$x_{\text{f}} = \frac{A^{\frac{1}{4}} M_{\text{f}} T_{\text{CMB}}}{B^{\frac{1}{4}}}, \tag{26}$$

into different parts. First, we set

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$$x_i = \frac{A^{\frac{1}{4}} M_i T_{\text{CMB}}}{B^{\frac{1}{4}}}, \tag{27}$$

and

$$\left(2arctan(x_f) + log\left(\frac{1 - x_f}{1 + x_f}\right)\right) - \left(2arctan(x_i) + log\left(\frac{1 - x_i}{1 + x_i}\right)\right) = \Delta t(4A^{\frac{3}{4}} B^{\frac{1}{4}} {T_{\text{CMB}}}^3). \tag{28}$$

resulting in the simplified version of the equation being

Then, x_f was found using MATLAB and then using Desmos to graph the function, $2_{arctan(xf)} + log(|1 - x_f|/(1 + x_f))$, and a line for $x = x_f$, a numerical value for that portion of the equation using the intersect point of the two lines. Next, a numerical value was found for $\Delta t(4A^{4/3}B^{1/4}T_{\text{CMB}}{}^3)$, using the number of seconds in the era and the temperature during the era, and then added onto the numerical value for $2_{arctan(xf)} + log(|1 - x_f|/(1 + x_f))$ to find y_i. Using Desmos, a line, $y = y_i$ was then graphed along with the function, $2_{arctan(xi)} + log(|1 - x_i|/(1 + x_i))$, to find x_i, which is the x value at the intersection point. Finally, M_i is found using MATLAB using the equation for x_i, finding the mass of the primordial black hole at the beginning of the era. This process is then repeated for each era and done for both the upper and lower bound of the mass range. The results were then compiled into two different tables with columns showing the different eras, the age of the universe at the beginning of each era, the temperature of the cosmic microwave background at that point in time, and the lower bound for the mass of a primordial black hole in Table 1 and the upper bound in Table 2.

Table 1

Lower Bound			
Era	Age of Universe	Temperature of CMB	Mass of Primordial Black Hole
End of Era 5	10^{10} years	3 K	$7.0000000000000000 \times 10^{13}$ kg
Beginning of Era 5	10^{9} years	10 K	$7.0000171636394049 \times 10^{13}$ kg
Beginning of Era 4	300,000 years	3000 K	$7.0000580014969614 \times 10^{13}$ kg
Beginning of Era 3	3 minutes	10^{10} K	$7.0000171636394054 \times 10^{13}$ kg
Beginning of Era 2	10^{-6} seconds	10^{13} K	$6.9999354879242937 \times 10^{13}$ kg
Beginning of Era 1	10^{-12} seconds	10^{15} K	$6.9999352879230690 \times 10^{13}$ kg

Table 2

Upper Bound			
Era	Age of Universe	Temperature of CMB	Mass of Primordial Black Hole
End of Era 5	10^{10} years	3 K	$8.000000000000000000000 \times 10^{18}$ kg
Beginning of Era 5	10^{9} years	10 K	$8.000136295180907986418 \times 10^{18}$ kg
Beginning of Era 4	300,000 years	3000 K	$8.000136295180908871947 \times 10^{18}$ kg
Beginning of Era 3	3 minutes	10^{10} K	$8.000136295180908344115 \times 10^{18}$ kg
Beginning of Era 2	10^{-6} seconds	10^{13} K	$8.000136295180908344115 \times 10^{18}$ kg
Beginning of Era 1	10^{-12} seconds	10^{15} K	$8.000136295180906917869 \times 10^{18}$ kg

Conclusion

The decrease in the temperature of the cosmic microwave background over time means that the absorption rate per square unit of a black hole decreases over time as the temperature of the cosmic microwave background is proportional to the absorption rate per square unit of a black hole according to the Stefan-Boltzmann law. There is also the fact that the Hawking radiation rate increases as the black hole's mass decreases as the black hole's mass is inversely proportional to the black hole's temperature, making it inversely proportional to the emission rate per square unit of a black hole. Therefore, the mass of a primordial black hole should increase in the very early universe until it reaches a point where the rate of mass the black hole is gaining from absorption of the cosmic microwave background is less than the rate of mass the black hole is losing from Hawking radiation. Then, the mass of the black hole should decrease until it fully evaporates. The results in Table 1 match this prediction, as it shows that both the lower and upper bounds of the mass range of the primordial black holes increased from era 1 to era 4 before decreasing during era 4 and 5. However, approximations were taken in calculating the results, such as using constant values for the cosmic microwave background temperature for each era and adjusting for the limitations of programs such as MATLAB and Desmos in finding exact numerical values. A perfect environment was assumed during the calculations, which does not consider the possibility of other objects falling into the black hole and affecting the mass. Nevertheless, the methods used in these calculations provide a reasonable estimate of the mass of primordial black holes over time that possibly could make up all of the dark matter in the universe.

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