

# Policy Making for Oil Imports by United Nations' Countries Using Multi-Objective Mixed Integer Linear Programming

Aarush Garg

German Swiss International School, No. 11, Guildford Road, The Peak, Hong Kong; gargaarush2006@gmail.com

Mentor: Dr. Benoît Legat

**ABSTRACT:** This paper highlights a method to assist policy-making for oil imports for United Nations member states to reduce their cost (including shipment) and pollution (including Carbon dioxide and Sulphur dioxide emissions). It assists with the transition towards the United Nations Sustainable Development Goal Number 7, of having a world with cleaner and affordable energy. Further, this paper discusses using mixed integer linear programming to solve a global optimal cost for oil transportation and ensure energy security while reducing the cost and pollution to the environment. Techniques such as Multiple Linear Regression, Branch and Bound, and the use of Primal and Dual values are used to solve this optimization problem. This paper attempts to provide a feasible and practical solution for policymakers. Future work can be done to enhance solutions to this problem, using certain aspects of Game Theory, such as the Nash equilibrium and shared utility, to ensure an optimal solution acceptable to all participating countries.

**KEYWORDS:** Energy: Physical Sustainable, Design Policymaking for United Nations, Mathematics Combinatorics, Mixed integer linear programming.

## ■ Introduction

Due to the emergence of increasing global temperatures and the accelerated consequences of global warming, it is of utmost importance to reduce the amount of greenhouse gases released by burning fossil fuels. Unfortunately, countries have not been able to harvest the full potential of renewable energy, as a statistic from the International Energy Agency<sup>1</sup> shows only 5.5% of global energy adoption is renewable energy, so to fulfill the energy demand posed by citizens of each country, their respective governments must fulfill this energy demand through other sources. According to the International Energy Agency, the most common energy source today is crude oil.<sup>2</sup> It accounts for 31% of total energy consumption; therefore, reducing the impacts of this commodity on the environment and making it a feasible solution cost-wise is highly important. Thus, this paper aims to solve this issue by transitioning to a world with sustainable energy sources while maintaining energy security for nations. It seeks to do so by aiding policymakers' decisions of where to import oil from to decrease the total cost (both shipping cost and price of oil) and the pollution cost (from carbon dioxide emissions in the refining process and Sulphur dioxide emissions while burning). As each country is attempting to transition to a sustainable and environmentally friendly world, it is of high priority for all member nations of the UN to come together to adopt policies to reduce the environmental impacts of their oil imports. Hence, this paper tackles the problem of minimizing global environmental impact and costs.

Mixed Integer Linear Programming is a technique based on the concept of Linear Programming with added constraints of some integer and non-integer variables. This technique solves various problems, such as scheduling problems, space optimization problems, and integrating renewable energy into energy

grids. One example of the use of MILP is in technician issues of scheduling and routing with many different constraints on tasks and skills, as discussed in Mathlouthi's paper.<sup>3</sup> Such MILP formulations commonly focus more on binary variables and are formulated based on physical constraints, such as working efficiency and time constraints. Another application is the optimization of space to maximize the energy output from wind turbines, as discussed in Kuo's paper<sup>4</sup> and Archer's paper.<sup>5</sup> This focuses more on constraints regarding space and location rather than binary variables. Another application of space optimization using MILP includes the integration of different renewable energy sources to maximize energy production, as discussed in Morais' paper<sup>6</sup> and in the paper by Alberizzi *et al.*<sup>7</sup> This is similar to the integration of renewable energy into small communities, which is discussed in Cosic's paper.<sup>8</sup>

Another aspect of sustainability that is solved using MILP is covered in Kantor's paper<sup>9</sup> and Ren and Gao's paper,<sup>10</sup> which discuss the application of MILP in the distribution of energy load and an integrated energy system. This paper focuses on energy integration in terms of sustainability objectives. The goal is to move the world toward a more sustainable path and align with the United Nations' Sustainable Development Goal Number 7: To have clean and affordable energy by 2030.

## ■ Methods

**Research Objective:** This section discusses the two main objectives this paper aims to solve and discusses some of the MILP techniques used to solve these problems.

### *Objective 1: Single Country Optimization:*

The first step towards solving the UN Sustainable Development Goal regarding clean and affordable energy is to optimize

the cost and pollution of oil imports for a single country. This can aid policymakers in deciding where to import oil from and how much oil to import from these nations. Nations would be fulfilling their responsibility to reduce their impact on the environment while maintaining cost budgets to ensure that the nation's energy demand is met. Additionally, this problem can easily be adapted and solved by adding constraints on which nations a country is a trading partner with or changing the objective function depending on the friendliness of the nation it is dealing with. Using this as a guiding tool, policymakers can more easily justify their decision on oil imports from nations. Hence, this paper provides a method to solve this problem, with room for tweaks to customize individual countries' objectives and constraints.

### Objective 2: Global Optimization:

A further expansion of this problem is explored when trying to solve for an optimal cost and pollution for all countries together. This is an even more important goal as it aligns even more closely with the United Nations SDG Number 7 and SDG Number 17, with all member nations uniting to work towards a common goal. This problem would use a similar formulation as the first problem of policymaking for one country; however, it would involve minimizing the cost and pollution of each country's choice of import rather than just one country's choice of import.

### Mixed Integer Linear Programming:

Usually, Mixed Integer Linear Programming (MILP) is implemented to solve optimization problems and to perform system analysis involving integer variables and convex constraints, often integrating many different techniques such as Robust Optimization or Bi-Level Optimization, Big M Formulation, Branch and Bound, and Multi-Objective Optimization to solve scheduling problems and the most efficient method to transfer resources, while serving some physical constraints (i.e. when transporting notebooks, you cannot transfer half a book).

As discussed in Chang *et al.*'s paper,<sup>11</sup> Mixed Integer Linear Programming has many advantages. Hence, given that the problem posed was convex and included physical constraints (i.e., the number of barrels of oil transported must be an integer), MILP was the most suitable technique to use. The advantages of this technique compared to linear programming and other optimization methods were many-fold, as listed below.

1. MILP considers variables that are bound by physical constraints and variables that are unbounded
2. The solution time of the MILP formulation is far less than other algorithms (computationally cheaper)
3. The method's power is immense, as it can deal with variables in matrices of significant sizes.
4. The dual and primal values prove the solution is feasible and optimal after using the branch-and-bound technique.

The optimizer calculates primal and dual values to ensure the problem is feasible. In a minimization problem, such as the one proposed, the primal is a minimization problem, and the

dual is formed by using non-negative Lagrange Multipliers to add further constraints to the objective function. This turns the dual problem into a maximization problem. The solution to the primal problem is an upper bound to the solution of the dual, and the dual is the lower bound to the solution of the primal. In specific convex problems that fall under constraint qualification conditions, the duality gap (difference between the primal and dual values) is zero, which indicates strong duality and gives the optimal solution to the minimization problem. The problem discussed in this paper is one such example of a problem with strong duality.

### Optimizer Used:

The problem is solved using the HiGHS optimizer, one of the powerful linear optimizers of the JuMP module,<sup>12</sup> in the Julia programming language, as shown in the code snippet below. This optimizer is most suitable for solving linear problems as it is explicitly built to optimize using linear constraints for convex optimization. By solving the primal and dual, it determines the feasibility of the solution. Moreover, the HiGHS optimizer is very good at dealing with matrices of variables and multiple constraints and has an easily definable and adaptable objective function. As this was a multi-objective optimization problem with convex variables and a large matrix size of roughly 200 x 200, it was ideal to use the HiGHS optimizer.

```
model = Model{HiGHS.Optimizer}
@variable(model, 0 <= transfer[a in exportingcountries, m in importingcountries], Int)

for i in importingcountries
    for e in exportingcountries
        if i == e
            @constraint(model, transfer[e,i] == 0)
        end
        @constraint(model, transfer[e,i] <= supply[e])
        @constraint(model, transfer[e,i] <= totaldemand[i])
    end
end

for i in importingcountries
    @constraint(model, sum(transfer[:,i]) == totaldemand[i])
end

for j in exportingcountries
    @constraint(model, sum(transfer[j,:]) <= supply[j])
end

@objective(model, Min, sum(transfer[m,n] * totalcost[m] for m in exportingcountries for n in importingcountries) +
    sum(transfer[m,n] * pollution[m] for m in exportingcountries for n in importingcountries) +
    sum(transfer .* selectiveshippingcost))
optimize!(model)
```

**Code Snippet:** The HiGHS Optimizer Code in Julia JuMP optimizes the cost and pollution objectives of crude oil imports and exports.

### Notations:

- $P(\alpha, \beta)$  = Price of Crude Oil (Dollars per Barrel)
- $\alpha$  = API Density
- $\beta$  = Sulphur Percentage
- $\gamma$  = Quantity of Oil per Barrel
- $\text{CO}_2(\alpha)$  = Carbon Dioxide Emissions (Pounds Per Barrel)
- $\text{SO}_2(\beta, \gamma)$  = Sulphur Dioxide Emissions (Pounds Per Barrel)
- $l$  = shipping distance between two countries
- $C(l)$  = cost (dollars per barrel)
- $k$  = Number of Grades of Oil per country
- $x$  = Importing country name (single country optimization)
- $S$  = Total number of exporters of crude oil
- $D$  = Total number of importers of crude oil (global optimization)
- $s$  = List of countries exporting oil
- $d$  = List of countries importing oil
- $w_c$  = Weight of Sulphur Dioxide Emissions
- $w_s$  = Weight of Carbon Dioxide Emissions

CO<sub>2</sub> = List of CO<sub>2</sub> values

SO<sub>2</sub> = List of SO<sub>2</sub> values

p = List of price of Oil per barrel

c = Matrix of shipping costs

q = Matrix of quantities of oil being exported and imported

A = Objective function for Single Country Optimization

B = Objective function for Global Optimization

W = set of all whole numbers

### Data Preparation:

The database used was the United Nations' energy statistics database. As shown in Figure 1, the oil imports and exports section was extracted from this database. The database also included the yearly consumption and production of oil; however, for this problem, the export and import data were used, as many countries may have had oil reserves from previous years or may have had a different policy on the total amount of imports and exports due to the market price of the grade of oil they are producing.

| Index  | country or area        | commodity transaction            | year | unit                  | quantity |
|--------|------------------------|----------------------------------|------|-----------------------|----------|
| 174206 | Algeria                | Conventional crude oil - imports | 2014 | Metric tons, thousand | 294      |
| 174231 | Argentina              | Conventional crude oil - imports | 2014 | Metric tons, thousand | 589      |
| 174256 | Aruba                  | Conventional crude oil - imports | 2014 | Metric tons, thousand | 8        |
| 174281 | Australia              | Conventional crude oil - imports | 2014 | Metric tons, thousand | 21835    |
| 174306 | Austria                | Conventional crude oil - imports | 2014 | Metric tons, thousand | 7510     |
| 174349 | Bahrain                | Conventional crude oil - imports | 2014 | Metric tons, thousand | 10385    |
| 174374 | Bangladesh             | Conventional crude oil - imports | 2014 | Metric tons, thousand | 1380     |
| 174407 | Belarus                | Conventional crude oil - imports | 2014 | Metric tons, thousand | 22508    |
| 174430 | Belgium                | Conventional crude oil - imports | 2014 | Metric tons, thousand | 32108    |
| 174455 | Bosnia and Herzegovina | Conventional crude oil - imports | 2014 | Metric tons, thousand | 934      |
| 174470 | Brazil                 | Conventional crude oil - imports | 2014 | Metric tons, thousand | 17757    |
| 174495 | Bulgaria               | Conventional crude oil - imports | 2014 | Metric tons, thousand | 5103     |
| 174520 | Cameroon               | Conventional crude oil - imports | 2014 | Metric tons, thousand | 1956     |
| 174534 | Canada                 | Conventional crude oil - imports | 2014 | Metric tons, thousand | 38614    |

**Figure 1:** United Nations dataset of all energy Exports and Imports. This is the raw dataset that will be processed, and from this, Crude Oil Exports and Imports will be extracted.

### Grades of Oil and Price Dependency on API Density and Sweetness - Multiple Linear Regression:

Different grades of oil, determined by the API Density and sweetness (Sulphur percentage), as shown in Figure 2, were used as metrics to distinguish the different types of oil each country produced. The price data was determined through multiple linear regression and extrapolation. This was done because there was a lack of oil price data online, and according to McKinsey Energy the price of oil is dependent on API Gravity and sweetness factors linearly. Thus, performing multiple linear regression and extrapolation could help give prices of all different oil grades.

The price of 12 different grades of oil, obtained from an oil website, were regressed against Sulphur percentage and API Density, using the least sum of squares criterion to help create a metric for the price dependence on API Density and Sulphur percentage. These 12 oil grades were chosen because they were the most stable, and the political factors and market characteristics affecting them were minimal. Hence, this extrapolation was used to predict the fair prices of other oil grades using the coefficients of API Density, Sulphur, and a constant term, as shown in Equation (1).

$$P(\alpha, \beta) = 135.57 - 0.66\alpha - 4.98\beta \quad (1)$$

| Crude            | API  | Sulfur | Source       |
|------------------|------|--------|--------------|
| Agbami           | 47.2 | 0.05   | Nigeria      |
| Akpo             | 45.8 | 0.07   | Nigeria      |
| Al Shaeen        | 30.3 | 1.90   | Qatar        |
| Amenam           | 37.0 | 0.17   | Nigeria      |
| Amna             | 37.0 | 0.17   | Libya        |
| ANS              | 31.4 | 0.96   | US           |
| Arab Extra Light | 40.0 | 1.09   | Saudi Arabia |
| Arab Heavy       | 28.0 | 2.80   | Saudi Arabia |
| Arab Light       | 33.0 | 1.77   | Saudi Arabia |
| Arab Medium      | 31.0 | 2.55   | Saudi Arabia |
| Arab Super Light | 51.0 | 0.09   | Saudi Arabia |
| Ardjuna          | 37.0 | 0.09   | Indonesia    |
| Attaka           | 43.0 | 0.09   | Indonesia    |
| Azeri Light      | 34.9 | 0.55   | Azerbaijan   |
| Bach Ho          | 39.0 | 0.04   | Vietnam      |
| Bakken           | 42.1 | 0.18   | US           |
| Banoco           | 31.8 | 2.45   | Bahrain      |

**Figure 2:** Crude Oil Grades with the different API Density and sulfur percentage. These are used to help determine the carbon dioxide and sulfur dioxide emissions from each barrel of oil.

➤ Use the following equation to calculate SO<sub>2</sub> emissions from fuel usage: SO<sub>2</sub> = S x F x 2

Where: SO<sub>2</sub> = sulfur dioxide emissions for the quantity of fuel burned, tons  
S = sulfur content of the fuel burned (fraction by weight)  
F = quantity of fuel burned, based on quantity measurement, tons  
2 = pounds of sulfur dioxide per pound of sulfur

**Figure 3:** The formula for SO<sub>2</sub> emissions uses sulfur percentage and quantity of oil burned. This helps determine the total amount of sulfur dioxide emissions from burning one barrel of crude oil.

### Approximating Sulphur Dioxide Emissions from Burning of Oil:

In this section, pollution from sulfur dioxide will be obtained using a formula for sulfur dioxide produced from burning oil with a certain sulfur percentage, as shown in Figure 3.

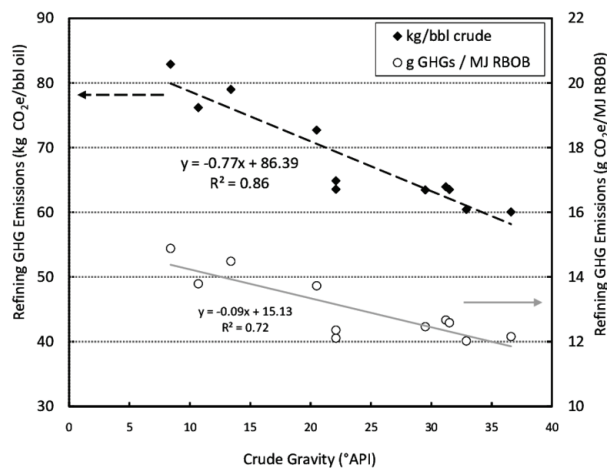
This is one of the two metrics used to model the impact of different oil grades on the environment.

$$SO_2 (\beta, \gamma) = 2\beta\gamma \quad (2)$$

### Approximating Carbon Dioxide Emissions in Refining of Oil:

A metric shown in Brandt's paper<sup>13</sup> and Figure 4 is used to estimate the carbon dioxide emissions produced in the refinery of oil given a specific API Density utilizing a method of linear regression with an R-squared value of 0.86. Using the coefficients given as an approximation, the CO<sub>2</sub> emissions produced in refining the crude oil can be calculated, as shown in (3), and used in combination with SO<sub>2</sub> emissions to serve as a metric for pollution caused by each barrel of oil.

$$CO_2 (\alpha) = 86.39 - 0.77\alpha \quad (3)$$



**Figure 4:** Approximation of  $\text{CO}_2$  emissions from refining crude oil against API Density using linear regression. This approximation helps to estimate the total carbon dioxide per barrel of oil.<sup>13</sup>

#### *Approximating Oil Statistics for Countries with Multiple Grades of Oil:*

Some countries, such as the United States of America, had multiple different grades of oil, and there was no data on the distribution of the oil percentages. Hence, an assumption was made that each grade of oil being exported was distributed equally. Thus, each country's price,  $\text{SO}_2$  emissions, and  $\text{CO}_2$  emissions were approximated as the mean of all oil grades from that country.

$$\frac{1}{k} \sum_{n=1}^k \text{CO}_{2n} \quad (4)$$

$$\frac{1}{k} \sum_{n=1}^k \text{SO}_{2n} \quad (5)$$

$$\frac{1}{k} \sum_{n=1}^k p_n \quad (6)$$

#### *Approximating Oil Statistics for Countries with No Data about Grades of Oil:*

Some countries had no data regarding the API density and sulfur percentage of the oil they were exporting. Hence, given the fact that Brent crude oil is the most common type of oil and the fact that these countries don't have their oil grade, it can be inferred that these countries have oil of similar API Density and sulfur percentage to Brent crude oil, so it can be approximated to have the same price,  $\text{CO}_2$  emissions and  $\text{SO}_2$  emissions as Brent crude oil. Hence, these statistics are now used for the optimization.

#### *Approximating Shipping Cost using Distance from Ports as an Indicator:*

Another statistic that had to be calculated was the shipping cost from each country to the other country in terms of cost per barrel of oil. Bertoli<sup>14</sup> approximated the distance from each country to all other countries via sea in nautical miles by assuming it to be between two major parts of both countries and then calculating the distance between these two ports. From expert discussions on various websites, it was approximated that the cost of transporting oil was 1 dollar per barrel per

1000 nautical miles. Hence, the data from the CERDI database was used to approximate transport cost as a function of sea distance through the formula below (7). Thus, a matrix for the cost of transportation per barrel of oil from each country to the other countries was obtained.

$$C(l) = \frac{l}{1000} \quad (7)$$

#### *Objective Function Discussion:*

Lastly, for the objective function, it is very important to give suitable weights to the two different metrics to minimize the objective function. On the one hand, countries want to reduce cost (shipping cost + cost of oil per tonne) while they also want to reduce the "cost" of pollution ( $\text{SO}_2$  emissions +  $\text{CO}_2$  emissions per tonne). However, given which objective they wish to prioritize, they will give this a higher weightage in the objective function. This weightage can be changed in the objective function by adding different coefficients to the different objectives to prioritize one objective over the other. There are a few different ways to choose these weights.

##### *Weights using Carbon Tax Policy:*

In this method, the weight for the pollution metric could be calculated using carbon tax and Sulphur dioxide removal costs to approximate carbon dioxide and Sulphur dioxide as costs. Then, the objective of reducing cost and pollution could be added, given that the units are the same. The value used as a weight for the carbon tax can be found in a study by the OECD,<sup>15</sup> and the value for Sulphur dioxide removal costs can be found in Burtraw's paper.<sup>16</sup> The use of carbon tax and credits has been a policy used worldwide and has been defined by the World Bank<sup>17</sup> as "a price on carbon by defining a tax rate on greenhouse gas emissions or – more commonly – on the carbon content of fossil fuel."

##### *Weights using Preference:*

In this method, the United Nations could use statistics regarding the cost of  $\text{CO}_2$  and  $\text{SO}_2$  emissions to society through their impact on the healthcare system, wildlife, and air quality to design a suitable metric and assign weights, which could be added to minimize the total objective function.

#### *Problem Formulation and Constraints:*

In this section, we will discuss the formulation of the problem, the constraints, and the objective functions for the different objectives.

##### *Goal 1: Problem Formulation for One Country's Policymaking:*

This section will discuss formulating the constraints and objectives for the single-country optimization problem.

##### *Constraints for Goal 1:*

This section contains two constraints for single-country optimization.

1. The first constraint shows that the quantity transferred of oil from one country to another must be a whole number (positive integer value) and less than the total oil supply from the exporting country, as shown below.

2. The second constraint shows that the sum of all imports of the country must be equal to the total demand of that country, as shown in (8)

$$\forall i \in s, 0 \leq q_x^i \leq s^i, \exists q_x^i \in W$$

$$\sum_{i=1}^S q_x^i = d^x \quad (8)$$

#### Objective Function for Goal 1:

This section highlights the objective function for the single country optimization, which can be simplified into three parts, as shown below.

Part 1: (Price of oil + Shipping cost of oil)  $\times$  Quantity Transfer of Oil

Part 2: Weight of  $SO_2 \times$  Quantity Transfer of oil  $\times SO_2$  emissions

Part 3: Weight of  $CO_2 \times$  Quantity Transfer of oil  $\times CO_2$  emissions

These three parts can be added together in the same units, as in (9) and (10). As discussed in the introduction section, the objective function can be easily tweaked to prioritize geopolitical situations and specific trends.

The HiGHS optimizer solves the minimization of this objective function.

$$\min A \quad (9)$$

$$i \in s$$

$$A = \sum_{i=1}^S q_x^i \cdot (p_i + c_x^i) + w_s(q_x^i)(SO_{2i}) + w_c(q_x^i)(CO_{2i}) \quad (10)$$

#### Goal 2: Problem Formulation for United Nations Policy Making:

This objective aims to collectively optimize all United Nations members' total costs and pollution.

##### Constraints for Goal 2:

There are three major constraints to this goal.

1. The sum of the quantity of oil transferred from one nation to all other nations must be less than or equal to the total amount of oil supplied by that one nation.

2. The sum of the quantity of oil transferred from all nations to one nation must equal the total demand for oil from that nation.

3. The quantity of oil transferred must always be a whole number (positive integer).

$$\forall i \in s, 0 \leq \sum_{n=1}^D q_i^n \leq s^i \quad (11)$$

$$\forall n \in d, 0 \leq \sum_{i=1}^S q_i^n = d^n \quad (12)$$

$$\forall a \in s, \forall b \in d, \exists q_a^b \in W$$

#### Objective Function for Goal 2:

The objective function can once again be split into three parts.

Part 1: (Price of oil + Shipping cost of oil)  $\times$  Quantity Transfer of Oil

Part 2: Weight of  $SO_2 \times$  Quantity Transfer of oil  $\times SO_2$  emissions

Part 3: Weight of  $CO_2 \times$  Quantity Transfer of oil  $\times CO_2$  emissions

These three parts form the objective function, which are added together. The only difference between this objective

function and the previous one is that it has multiple demanding nations; hence, it will also have to be summed over all the demanding nations.

Once again, tweaks can be made to this objective function depending on certain external factors in certain countries, but this serves as the base function, including all the essential costs.

$$\min B \quad (13)$$

$$i \in s, n \in d$$

$$B = \sum_{i=1}^S \sum_{n=1}^D q_i^n \cdot (p_i + c_i^n) + w_s(q_i^n)(SO_{2i}) + w_c(q_i^n)(CO_{2i}) \quad (14)$$

## ■ Result and Discussion

This section will review and analyze the results of optimizing the two objective functions while discussing key topics such as game theory, choosing weights, and using integer variables.

#### Objective 1: Minimising Cost and Pollution from Crude Oil to a Member Nation:

In this section, we will discuss the results from various countries, the importance of shipping cost on the results, the implementation of the carbon tax and sulfur dioxide replacement to give weight to the pollution index in the objective function, and the results from this.

##### Discussion of Results from India and China:

As seen from the sample table above, to ensure the optimal policy for oil imports from other countries, the constraints set in the problem formulation must be met, and the objective function must be minimal. The results are very similar, as shown in Tables 1 and 2, given that most features remain constant when changing from India to China. For example, the  $CO_2$  emissions,  $SO_2$  emissions, and the Price of Oil per barrel stay the same. The only things that change are the shipping cost and the quantity of oil demanded for import. Additionally, given that China and India are very close in distance, shipping cost differences should be minimal.

##### Dependence of Results on Distance - Shipping Cost:

If a third country, such as the United States, is chosen, it may give very different results to these two countries noted above, given that the shipping distance from other nations varies largely. Recall that the shipping cost between two countries can be calculated using (7). Furthermore, as discussed in the methodology, if coefficients are added to each objective to weight one more than the other, this will change the solution. For example, suppose the environmental pollution is given a coefficient of 10 times greater than the cost. In that case, results will be skewed towards importing from countries with the most clean oil. If cost is given a higher coefficient, then results will be skewed towards obtaining the cheapest oil possible without considering the impact on the environment significantly.

##### Discussion of Carbon Tax and Sulphur Dioxide Cost to choose weights of pollution:

A solution to choosing the optimal weights would be to use the carbon credit or carbon tax system as a metric to estimate the impact of Carbon Dioxide emissions on each nation and to estimate the weight of Sulphur dioxide as the cost of reducing Sulphur dioxide emissions per tonne. These weights give a rea-

sonable estimate of the "cost" of carbon emissions, and hence, the pollution metric and cost of shipping and oil per barrel can be added together as they have the same units.

**Table 1:** Results of China's optimization: import policy to optimize the cost and pollution of oil transfer for China.

| Country Name      | Barrels of Oil |
|-------------------|----------------|
| Nigeria           | 756294         |
| Norway            | 469647         |
| Kazakhstan        | 223919         |
| Algeria           | 172621         |
| Equatorial Guinea | 99037          |
| Malaysia          | 83796          |
| Australia         | 81355          |
| Gabon             | 81297          |
| Gabon             | 79105          |
| Vietnam           | 68212          |
| Ghana             | 39480          |
| Chad              | 36940          |
| Denmark           | 35176          |
| Argentina         | 14506          |
| Sudan             | 13106          |
| Papua New Guinea  | 5891           |

**Table 2:** Results of India's optimization: import policy to optimize the cost and pollution of oil transfer for India.

| Country Name      | Barrels of Oil |
|-------------------|----------------|
| Norway            | 469647         |
| Algeria           | 172621         |
| Nigeria           | 108368         |
| Equatorial Guinea | 99037          |
| Malaysia          | 83796          |
| Australia         | 81355          |
| Gabon             | 81297          |
| Vietnam           | 68212          |
| Ghana             | 39480          |
| Chad              | 36940          |
| Denmark           | 35176          |
| Argentina         | 14506          |
| Sudan             | 13106          |
| Papua New Guinea  | 5891           |

### **Objective 2: Minimising Global Cost and Pollution from Crude Oil as a United Nations Policy Maker:**

In this section, we will discuss the results of the global optimization problem, the implementation of game theory, and the use of integer vs. decimal variables.

#### *Discussion of How to Interpret Results:*

The table above, as shown in Table 3, is a sample of the solution for optimizing the transfer of oil from one country to another. Each column represents the total amount of oil the importing country takes from each exporting country. For example, in the first column, as shown in Table 3, Aruba imports 2047 barrels of crude oil from Albania and no oil from the remaining countries in this sample.

**Table 3:** Results of Global Optimisation: import policy to optimize global cost and pollution.

| Country Exportin  | Country Importing      | Barrels of Oil |
|-------------------|------------------------|----------------|
| Albania           | Aruba                  | 2047           |
| Albania           | Bosnia and Herzegovina | 224141         |
| Albania           | Belarus                | 6504           |
| Argentina         | Switzerland            | 13621          |
| Australia         | Brazil                 | 24614          |
| Belarus           | Belgium                | 11852          |
| Brazil            | Austria                | 151851         |
| China             | Belgium                | 4399           |
| Czechia           | Austria                | 197            |
| Equatorial Guinea | Brazil                 | 99037          |

#### *Discussion of Integers vs Decimal Variables:*

Another factor to discuss is the implementation of integer variables vs relaxing the constraint of integers for the JuMP function. As seen from the below summary, the solve time for integer constraints is double the time for decimal constraints. In addition, both strategies are just indicative measures for a country's imports. While, in reality, a country cannot import a fraction of a barrel of oil, it may not practically be able to import an integer, which the algorithm suggested, because a ship transporting the oil may only be able to facilitate a certain amount of oil. The fixed cost of running another ship is too large to justify importing oil from that country. The objective function is altered by less than 0.1 percent when changing from decimal to integer. Hence, both solutions are equally useful indicative measures of the optimal supply. As discussed in Leiserson and Saxe's paper,<sup>18</sup> the solve time for a MILP formulation with integer constraints is longer than that without integer constraints.<sup>10</sup> Given that both solutions are similar, and this is only an indicative measure, the decimal solution could also be used, as it is easier for the HiGHS optimizer to solve and is computationally cheaper.

## **Conclusion and Scope for Future Work**

### **Discussion of Game Theory – Nash Equilibrium and Shared Utility:**

The optimizer in the United Nations optimization problem solves a global optimum solution, which is a massive step for the members of the United Nations by keeping the cost and pollution to a minimum. However, to ensure that the cost is distributed equitably, applying the Nash equilibrium or another constraint is important to penalize large differences in cost and pollution between 2 or more countries. This will be a further step towards making the world more equal, and though the objective cost may be fractionally higher, all countries would think of this as a fair solution, and policymakers would be contented to implement such a strategy. The concept of shared utility is a different approach to solving the problem of the large disparity in objective functions between 2 or more coun-

tries. Countries with lower objective costs and pollution could share a percentage of their payoff with less advantaged nations to ensure that all nations are satisfied with this solution and that the global optimum is the best solution for all countries. The global optimum, however, is a large step towards achieving clean and affordable energy, as all members of the United Nations collectively are reducing environmental pollution. This reduction in total costs makes it sustainable and, hence, a long-term feasible solution.

### ***Difference in Objective Value for each country for Global Optimization vs Optimization for a single country:***

This section adds to the previous section by finding a unique solution to provide a country with an incentive or disincentive to ensure that each country follows the global solution provided. Either country can be given a payoff to use the concept of shared utility to help and make all stakeholders happier, or the UN can implement a tariff or levy for all countries that do not follow the policy, which increases their total cost beyond the optimum provided in the global optimization.

As stated previously, the Nash Equilibrium should be found to make it the fairest distribution rather than the most optimal, or there should be a constraint to squeeze the difference between objective costs for each country. However, in this paper, the global optimum solution is being found, as this shows how each country should proceed to buy oil. The distribution of profit (shared utility) is a further step to make this more equitable. However, the total global cost stays the same.

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### **Author**

Aarush is 17, a passionate mathematics believer in UN Sustainable Development Goal 7, and finishing his junior year in high school. As someone passionate about mathematics and energy research, Aarush wishes to explore concepts of number theory, algebra, and mathematical optimization to contribute to a cleaner and better world for future citizens. Outside of school, Aarush is a passionate cricket player and recently represented the Hong Kong Under 19 Cricket Team in the ICC Cricket World Cup Qualifier.