

The Earth's Rotation is Slowing Down: The Physical Reasons and How It Affects the Number of Days in a Year

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ABSTRACT: The tidal force of the Moon on the Earth can slow down the Earth's rotation, forming a "locked" or tidal synchronization phenomenon. Over the long course of the Earth's history, the Moon has gradually moved away from the Earth, and the energy of the Earth's rotation has been dissipated by the tidal forces on the surface and inside of the Earth, causing the Earth's rotation to slow down, resulting in an increase in the length of the day and a decrease in the number of days in the year. This study used Newton's law of universal gravitation and the law of conservation of angular momentum to derive a calculation formula for the rate at which the Earth's rotation slowed down, estimated the number of days in a year hundreds of millions of years ago, and verified the accuracy of the estimate using evidence from the Devonian tetracoral ridges.

KEYWORDS: Physics and Astronomy, Astronomy and Cosmology, Earth Rotation, Tidal Locking, Rugose Corals.

■ Introduction

Tides are the periodic rise and fall of ocean water caused by the gravitational pull of the Moon and the Sun. The Sun's gravitational pull on the Earth is almost 200 times stronger than the Moon's. However, the Moon's tidal effect is greater because the Moon's distance from the near side to the far side varies much more than the Sun's. The tidal force stretches the Earth along the line between the Earth and the Moon. Different points on the Earth's surface are at different distances from the Moon. Places facing the Moon experience a greater gravitational pull, and the ocean water bulges outward. On the far side, the Moon's pull on the Earth is greater than on the water, also causing high tides. Therefore, the two directions of the ocean surface perpendicular to the line connecting the Earth and the Moon correspond to low tides. As a result, the sea level rises and falls about twice a day.¹

The growth of any organism on Earth is affected by environmental factors. Over the long years, as the Earth revolves around the sun, it produces seasons and tides. These changes have a periodic impact on the growth of the hard parts and bones of organisms.

Corals produced thickened growth bands once per year, like tree rings, due to seasonal changes in water conditions. Some scientists, using the isotope Calcium-45, show that in reef corals, the rate of calcium carbonate uptake in coral tissues falls at night or in darkness and rises during the day. John W. Wells from Cornell University investigated the effects of this on the coral skeleton. He observed there were tiny ridges that may be deposited daily throughout the life of the coral, as shown in Figure 1. In 1963, Wells published his landmark paper titled "Coral Growth and Geochronometry" in the journal, *Nature*. He counted the smallest ridges on modern corals and discovered that there were about 360 per year. This strongly suggests that the growth lines are diurnal or circadian in nature.² He then counted these growth lines on fossil corals and discovered

that there were about 402 ridges on corals from the Silurian (430 million years ago) and 377 ridges on corals from the Jurassic (180 million years ago). These discoveries implied that the number of days in a year has been reduced over geologic time.³

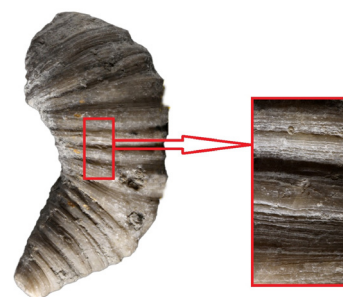


Figure 1: The Devonian rugose coral, with its fine growth lines in yearly skeletal deposition. Each fine growth line represents a day and night. By counting the number of growth lines included in each growth wrinkle the number of days in a year can be inferred. (credit "photo": modification of work by the Field Museum of Natural History).

In this study, we first made a theoretical analysis of how tidal force affects the Earth's rotation, based on Newton's law of universal gravitation and conservation of angular momentum of the Earth-Moon system. The change rate of the Earth's rotation was derived from which we estimated how many days there were in a year hundreds of millions of years ago. The calculated results were compared with those found by Wells in rugose corals, verifying the rationality of the calculation model.

■ Methods

If the Earth-Moon system is approximately regarded as an isolated system, its total angular momentum is conserved,

$$L_{tot} = I_E \omega_E + I_M \omega_M + I_{orbit} \omega_{orbit} \quad (1)$$

where L_{tot} is the sum of the angular momentum of the Earth-Moon system; I_E , I_M and I_{orbit} are the moments of inertia

respectively; ω_E , ω_M , and ω_{orbit} are the angular velocity of the Earth's rotation, the Moon's rotation, and the Moon's orbit, respectively.

The total kinetic energy of the Earth-Moon system is

$$E_K = \frac{1}{2}(I_E\omega_E^2 + I_M\omega_M^2 + I_{orbit}\omega_{orbit}^2) \quad (2)$$

From Eqns. (1) and (2), we can get the Cauchy-Schwarz inequality:⁴

$$2E_K(I_E + I_M + I_{orbit}) \geq L_{tot}^2 \quad (3)$$

When $\omega_E = \omega_M = \omega_{orbit}$, the minimum total kinetic energy can be obtained:

$$E_{K,min} = \frac{L_{tot}^2}{2(I_E + I_M + I_{orbit})} \quad (4)$$

When this happens, the Earth-Moon system moves like a rigid body, a phenomenon called “locking” or tidal synchronization. In this case, the Earth-Moon system only moves as a whole, without relative motion, so the total kinetic energy is minimal. It has already happened to most moons in our solar system, including Earth's Moon. The Moon currently is always showing the same side to the Earth, and therefore its rotation period has locked into the orbital period about Earth. The same process is happening to Earth, and eventually, the Earth will become tidally locked to the Moon. If that does happen, we would no longer see tides, as the tidal bulge would remain in the same place on Earth, and half the Earth would never see the Moon. However, this locking will take many billions of years, perhaps not before our Sun expires.¹

The Moon has an elliptical orbit in which the value of R varies by just over 10 %. It is reasonable to assume the Moon's orbit is circular. The I_E , I_M , and I_{orbit} in Eq. (1) can be calculated by

$$I_E = \frac{2}{5}M_ER_E^2 \quad (5)$$

$$I_M = \frac{2}{5}M_MR_M^2 \quad (6)$$

$$I_{orbit} = M_MR_{orbit}^2 \quad (7)$$

where the mass of Earth (M_E) is 5.97×10^{24} kg, the mass of Moon (M_M) is 7.35×10^{22} kg, the average radius of Earth (R_E) is about 6370 km, and the radius of Moon (R_M) is about 1700 km, and the average distance between the centers of Earth and the Moon (R_{orbit}) is 3.84×10^8 km.¹ The calculation results show the moment of inertia of the Moon's orbit is about 1.28×10^5 times of the Moon's rotation, and because the Moon's rotation has the same angular velocity as the Moon's orbit ($\omega_M = \omega_{orbit}$), thus the angular momentum of the Moon's rotation is negligible in Eq. (1).

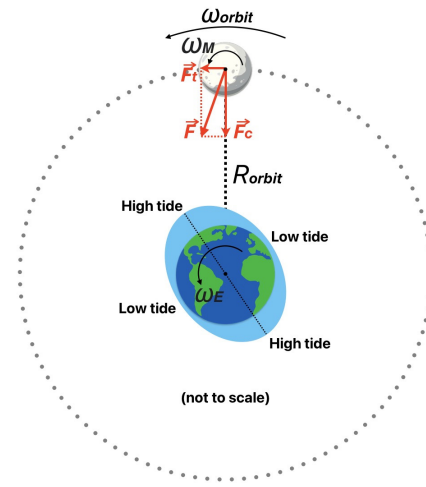


Figure 2: The high tides and low tides and the Earth-Moon gravitational force on the Moon. The high tide level on the ocean surface is not on the line connecting the Earth and the Moon, which makes the gravitational force of the Earth on the Moon slightly point toward the high tide level.

Earth rotates approximately every 24 hours. Since the Moon orbits Earth approximately every 27.3 days, and in the same direction as Earth rotates, Earth rotates much faster than the Moon orbits Earth ($\omega_E \gg \omega_{orbit}$). This causes the high tide level on the ocean surface not to be on the line connecting the Earth and the Moon, but slightly offset in the direction of the Earth's rotation, as shown in Figure 2. The shape of the ocean surface makes the mass distribution of the Earth no longer spherically symmetric. The gravitational force of the Earth on the Moon is no longer along the Earth-Moon line, but slightly points to the high tide level, as shown in Figure 2. The Earth-Moon gravitational force has a tangential component to accelerate the orbital motion of the Moon. This acceleration of the Moon maintains the conservation of total angular momentum, while also causing the Moon's orbit move away from the Earth. The Moon is not moving away from the Earth at a constant speed. This speed will change over time and is affected by many factors. The Earth will eventually become tidally locked to the Moon after many billions of years, at which point the distance between the Moon and Earth will no longer change. Through the action of tidal forces, the angular momentum of Earth's rotation is slowly being transferred to the Moon's orbit, as described by

$$\Delta L_E = -\Delta L_{orbit} \quad (8)$$

The Moon's orbital speed can be obtained by,¹

$$v_{orbit} = \sqrt{\frac{GM_E}{R_{orbit}}} \quad (9)$$

where $G = 6.67 \times 10^{-11}$ Nm²/kg² is the universal gravitational constant. Then the orbital angular momentum of the Moon around the Earth is

$$L_{orbit} = M_M v_{orbit} R_{orbit} = M_M \sqrt{GM_E R_{orbit}} \quad (10)$$

Then

$$\frac{\Delta L_{orbit}}{L_{orbit}} = \frac{1}{2} \frac{\Delta R_{orbit}}{R_{orbit}} \quad (11)$$

The angular momentum of the Earth's rotation is

$$L_E = I_E \omega_E \quad (12)$$

Then

$$\frac{\Delta L_E}{L_E} = \frac{\Delta \omega_E}{\omega_E} \quad (13)$$

From Eqns. (8), (11), (12), and (13) we get

$$\Delta \omega_E = -\frac{L_{orbit}}{2I_E} \frac{\Delta R_{orbit}}{R_{orbit}} \quad (14)$$

The Moon is spiraling away from Earth at an average rate of 3.8 cm per year, as detected by the Lunar Laser Ranging experiment.⁵ As a comparison, the Earth spirals away from the Sun at a rate of about 1.5 cm every year, causing its orbital speed to drop by about 3 nanometers per second over that timescale.⁶ Since the Sun-Earth system is much larger than the Earth-Moon system, the time it takes Earth to revolve around the Sun is generally considered constant in the units of hours or seconds, but not constant in days because the Earth's rotation speed is changing.

■ Results and Discussion

A day is the time it takes the Earth to make one revolution about its axis or to spin once around, so now a day is becoming longer than before, or now a year has fewer days than before. Based on the analyses above, we can estimate how many days were in a year when these corals were growing. Now the angular velocity of the Earth's rotation is

$$\omega_{E,now} = \frac{2\pi}{24[\text{hours}] \times 60[\frac{\text{minutes}}{\text{hour}}] \times 60[\frac{\text{seconds}}{\text{minute}}]} = 7.27 \times 10^{-5} \text{ [s}^{-1}\text{]} \quad (15)$$

Over the past 180 million years, the change in radius of the Moon's orbit can be calculated by

$$\Delta R_{orbit} = 0.038 \left[\frac{\text{m}}{\text{year}} \right] \times 180 \times 10^6 [\text{year}] = 6.84 \times 10^6 [\text{m}] \quad (16)$$

From Eqns. (10), (14), and (16), the change in angular velocity of the Earth's rotation can be obtained:

$$\Delta \omega_E = -2.65 \times 10^{-6} \text{ [s}^{-1}\text{]} \quad (17)$$

Thus, 180 million years ago, the angular velocity of the Earth's rotation was

$$\omega_{E,past} = \omega_{E,now} - \Delta \omega_E = 7.54 \times 10^{-5} \text{ [s}^{-1}\text{]} \quad (18)$$

Now, a typical year has 365.25 days.¹ Let N represent the number of days that a year had 180 million years ago, then

$$\frac{N}{365.25} = \frac{\omega_{E,past}}{\omega_{E,now}} \quad (19)$$

From Eqns. (15), (18), and (19), we find that a year had 378.5 days 180 million years ago. This has good agreement with the findings of the ridges on corals (377 days per year, 180 million years ago). Table 1 lists our calculation results in comparison to those from counting growth lines on fossil corals for three different geologic times.^{3,4} Our estimation errors are within $\pm 2.0\%$.

als for three different geologic times.^{3,4} Our estimation errors are within $\pm 2.0\%$.

Table 1: Estimate of how many days in a year millions of years ago from counting ridges on corals and from the calculation based on tidal force effects on the Earth's rotation.

Geological time	Days in a year from counting ridges on corals	Days in a year from our calculation	Error [%]
180 million years ago	377	378.6	0.43
370 million years ago	400	393.0	-1.74
430 million years ago	402	397.6	-1.08

Wells mentioned in his paper that throughout Earth's history, a slowdown of about 2 seconds per 100,000 years has occurred according to recent estimates.² At the beginning of the Cambrian, the length of the day would have been 21 hours. Based on this information, he developed a simple relation between the geological time scale and the number of days per year. We cited his data in Table 2 and made our calculation in comparison to his results. Our calculation results are smaller than Wells' estimates, and the error increases with geological time. The largest calculation error, -3.10 %, nevertheless, occurred in the Precambrian period, which is about 600 million years ago (Figure 3). We do not adopt Wells' method in calculation because now the Lunar Laser Ranging experiment detects that the Moon is spiraling away from the Earth at an average speed of 3.8 cm per year. This gives another linear relationship with a smaller slope. Both lines in Figure 3 start from 365 days per year. This leads to our calculated results are always being smaller than Wells' results, and the difference becomes larger and larger as the geological age increases. Our calculations are more precise because they are based on observations from the more sophisticated Lunar Laser Ranging experiment.

Table 2: Number of days per year in past geological time: comparison between J. Wells' estimates and our calculation.

Geological period	Geological time [millions of years]	Days in a year, data from J. Wells, 1963	Days in a year from our calculation	Error [%]
Cenozoic-Cretaceous	65	371	370.1	-0.25
Jurassic	135	377	375.3	-0.46
Triassic	180	381	378.6	-0.62
Permian	230	385	382.4	-0.68
Pennsylvanian	280	390	386.2	-0.98
Mississippian	312	393	388.6	-1.12
Devonian	345	396	391.1	-1.23
Silurian	405	402	395.7	-1.56
Ordovician-Cambrian	500	412	403.1	-2.17
Precambrian	600	424	410.9	-3.10

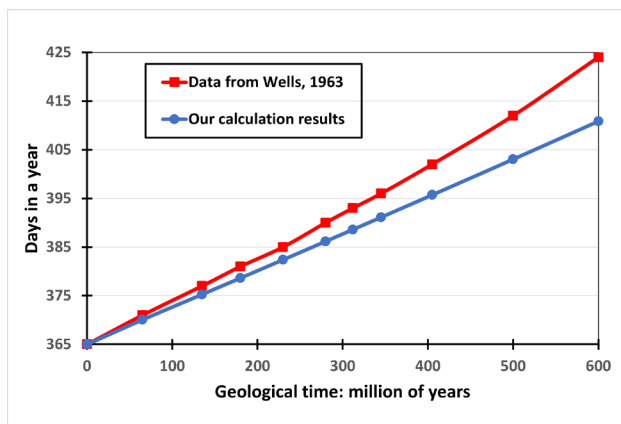


Figure 3: Relation between days in a year and geological time, comparison between J. Wells' results and our results. The difference increases with geological time, but in an acceptable range, with the largest calculation error of 3.10 %.

■ Conclusion

The tidal force of the Moon on the Earth slows down the Earth's rotation, eventually forming a phenomenon called "locking" or tidal synchronization. Throughout the history of the Earth, the Moon has been gradually moving away from the Earth. As the tidal forces on the Earth's surface and interior consume the energy of the Earth's rotation, the Moon's rotation slows down, resulting in a continuous increase in the length of the day and a continuous decrease in the number of days in a year. Using the Lunar Laser Ranging Experiment, researchers have determined that the Moon is moving away from the Earth at a rate of 3.8 cm per year. We assume that the orbit of the Moon is circular and the Earth-Moon system is considered an isolated system. We use Newton's law of gravity and the law of conservation of angular momentum to derive an equation that can calculate the rate of deceleration of the Earth's rotation. It can estimate how many days there were in a year hundreds of millions of years ago, but how to test this estimation over such a long time frame is a challenge. The growth of any organism on Earth is constrained by environmental factors. Over the long years, as the Earth revolves around the Sun, the seasons change, and the tides rise and fall, these changes have a periodic impact on the growth of the hard body and bones of organisms. The alternation of day and night in a day has a great impact on organisms, and this law must have a significant impact on organisms. Correspondingly, ancient organisms will show traces of regularity due to the alternation of day and night, and these effects are generally reflected in the hard parts of the organisms. These patterns could help determine Devonian storm frequency, changes in coral growth conditions, and potentially detect astronomical cycles like Earth's rotation rate and tidal periodicity.⁷ Therefore, the fossils preserved by these organisms are the best ancient calendars, also known as "paleobiological clocks." Paleontologists have discovered that the calcium carbonate secreted by coral polyps every day can form tiny "day rings" that gather into a growth belt that is significantly different from other years, just like the annual rings of a tree. Scientists can determine how many days there are in a year based on the number of "day rings" in the growth belt. This prompted us to use experimental data

from ancient corals to verify our hypotheses and computational models, and to use evidence from Devonian tetracoral ridges to verify the accuracy of our estimates. The calculation results are consistent with the actual observational data.

Many factors affect the Earth's rotation. In addition to the tidal force of the Moon, there are disturbances in the Sun's gravity, changes in the Earth's mass caused by cosmic dust, crustal movement, glacier melting, earthquakes, etc. The temporal variations in the atmospheric angular momentum and their contribution to the instabilities of the Earth's rotation are also non-negligible.⁸ The decadal rotational variations likely arise from gravitationally driven electromagnetic coupling between inner and outer cores and the mantle.⁹ These factors work together to cause small changes in the speed and direction of the Earth's rotation. In this paper, we show that it is possible to derive a predictable method to estimate how many days there were in a year in the geological past.

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