

Constant Dark Energy Density in the Universe on a Large Scale: A Challenge to Energy Conservation

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ABSTRACT: A mysterious force termed dark energy is believed to accelerate the expansion rate of the universe. Dark energy is assumed to have a constant density, which appears to violate the energy conservation principle, a universal law of physics, on large scales. While Noether's Theorem may explain the constant density of dark energy as the universe expands, many other frameworks, including the backreaction hypothesis, are still under investigation. Researchers have tried to understand dark energy through the application of several aspects of physics, including quantum field theory and general relativity. In this review, we suggest that Noether's theorem may provide a plausible explanation to resolve this by connecting energy conservation to space-time symmetry. This challenges the notion that regular energy conservation is a universal law that can be applied to all scales, irrespective of other factors. This research is important as it helps to deepen our understanding of dark energy, ultimately helping us to figure out the mystery of cosmic evolution. In this review paper, we explore the quantum approach to explain this density, followed by an alternative to dark energy through backreaction in the universe, concluding with Noether's Theorem, which is the most plausible explanation given to date.

KEYWORDS: Physics and Astronomy, Astronomy and Cosmology, Dark Energy, Energy Conservation, Noether's Theorem.

■ Introduction

The universe expands in a puzzling manner, as galaxies seem to drift away, pushed by an unknown force. In the late 1990s, it was discovered that the expansion rate of the universe is increasing. When the luminosity distances of Type Ia supernovae were measured, and the data from the cosmic microwave background and baryon acoustic oscillations were analysed, it was confirmed that the acceleration was being driven by a mysterious component that was termed 'dark energy'.¹ Dark energy was a new form of energy that showed an effect similar to repulsive gravity. Although it was initially discovered from observations of distant supernovae, dark energy is now supported by observations from various cosmological structures, such as cluster abundance² and weak gravitational lensing.³ It has also been measured that the universe is composed of 68% dark energy, 27% dark matter, and only about 5% ordinary matter.⁴

Although the cosmological constant was initially introduced in Einstein's theory of General Relativity to allow for a static universe, it was later abandoned after it was discovered that the universe was expanding. It was later reintroduced in the Λ CDM model to explain the accelerating expansion of the universe.⁵ But this has led to inherent theoretical problems, such as the large difference between the value of dark energy that was observed and the predictions from Quantum Field Theory.⁶ Such problems may suggest that there are some interactions of dark energy that have not been discovered.

The most widely accepted theory in cosmology is the Λ CDM model. In this framework, dark energy serves as a cosmological constant Λ , which has a fixed density that does not change as the universe expands. However, this behaviour of dark energy conflicts with the notion that all forms of energy must be

conserved in the universe. This paradox comes into existence when conventional energy conservation principles are applied across cosmological scales, when other theories of physics, such as general relativity and quantum field theory, began to diverge.

The notion that dark energy density remains constant when the universe expands raises several fundamental questions about the applicability of the traditional principle of energy conservation. This could be an indication of the possibility that classical conservation laws break down at cosmological scales. This could also mean that our current models and principles are lacking. Several theories and principles based on symmetry have now been suggested to solve this paradox. Among them, Noether's Theorem has emerged as a key candidate for rethinking our current principle of energy conservation.⁷ Noether's Theorem connects conservation principles to the symmetries of spacetime. The observed constant density of dark energy is not a paradox if Noether's symmetries can be applied to an expanding universe. Rather, it could be simply an outcome of space-time geometry and symmetry. Solving this paradox could help us further understand our universe.

This review paper suggests that the application of Noether's Theorem could explain the constancy of dark energy in the expanding universe based on symmetry. The explanation based on Noether's Theorem is simpler and more plausible than other explanations given to solve this problem. This paper is organized into five sections. It begins with a section on the discovery and background of dark energy. It also talks about the relation between dark energy and the conservation laws. The second section discusses the role of dark energy as the cosmological constant in the Λ CDM model. This is followed by a section talking about the quantum and relativistic approach-

es to explain the constant density of dark energy. We talk about the process of backreaction, which explains acceleration without dark energy in the fourth section. The fifth and final section talks about Noether's Theorem and how it attempts to solve this paradox in a very logical manner.

In this paper, we will discuss solutions to the question: How can the constant density of dark energy be explained by the energy conservation principle in an expanding universe?

■ Discussion

Section 1 - Background of Dark Energy and Conservation

Laws:

This section reviews the discovery and behaviour of dark energy. It talks about the conflict of constant dark energy density and traditional conservation of energy, as well as the mismatch between theory and observation.

The analysis of observations of distant Type Ia supernovae in the late 1990s revealed something that was unexpected. It indicated that the expansion of the universe is accelerating.⁸ Considered as "standard candles" because of their predictable luminosity, these supernovae helped astronomers to measure distances between bodies in the universe. It was observed that galaxies were moving away from each other at a rate that was speeding up. This was surprising to astrophysicists as gravity, which attracts matter together, was expected to slow down the rate of expansion. This was the exact opposite of what was observed. The acceleration of the expansion rate alluded to the presence of a repulsive force or energy throughout the empty universe.

The idea of this repulsive force was also supported by observations from various other data sources. Complementary data from the cosmic microwave background (CMB) and large-scale structures confirmed the presence of a repulsive force that has negative pressure.⁴ The CMB is the oldest light in the universe. It is the remnant microwave radiation after the Big Bang. Detailed information about the geometry and composition of the universe can be obtained from tiny fluctuations in the CMB. Satellites like Planck and WMAP provided data that showed us that although the universe is spatially flat on average, its rate of expansion is increasing. This gave us an indication of the presence of an energy form that exerted negative pressure. The large-scale surveys of the distribution of galaxies also supported this. These observations hence confirmed the presence of a mysterious force or energy now termed as dark energy. Dark energy is estimated to contribute to about 70% of the total energy composition of the universe.⁴

These conclusions proved to be a challenge to some of the integral properties of physics, such as the law of conservation of energy. To further comprehend this, we must further examine the conservation of energy in the context of general relativity. In general relativity, a globally conserved energy is not always defined, as our cosmological models lack global time-translation symmetry.⁹ As a result, general relativity does not have a universal law of energy conservation, unlike in Newtonian mechanics.¹⁰ This is because gravity is described as the curvature of spacetime, which evolves with the energy and matter

the universe contains. This results in a fundamental mismatch regarding a basic principle in the two major theories of physics.

In general relativity, energy conservation is expressed as the disappearance of the covariant divergence of the energy-momentum tensor in a local context:¹¹

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad (1)$$

In this equation, ∇_{μ} represents the covariant derivative, which accounts for the curvature of spacetime. $T^{\mu\nu}$ is the energy-momentum tensor, which is responsible for describing the density and flow of energy and momentum. This equation tells us that although energy and momentum are conserved in the universe locally, that is, in any small region of spacetime, it does not suggest that the total energy in the entire universe is globally conserved. Therefore, it becomes essential to formulate a global energy conservation law that applies to an expanding universe.¹²

Dark energy is included in the Λ CDM model as the cosmological constant Λ .⁶ In classical physics, the energy of baryonic matter and radiation spreads out as the universe expands. But dark energy, instead of spreading out, increases in amount to fill the infinitely growing volume of the universe due to expansion.¹³ This violates the conventional law of conservation of energy as it suggests that the total amount of energy is not fixed, which conflicts with the Newtonian notion that energy cannot be created or destroyed.¹⁴

The paradox of the constant dark energy density with respect to the energy conservation law is a large problem in theoretical physics, as it suggests that the conventional energy conservation principle may not be global and may not work in an expanding, curved universe. This may suggest a breakdown of classical physics at all scales, especially quantum scales. It also highlights the need for modifying our understanding of the laws of physics so that they are applicable on a universal scale.

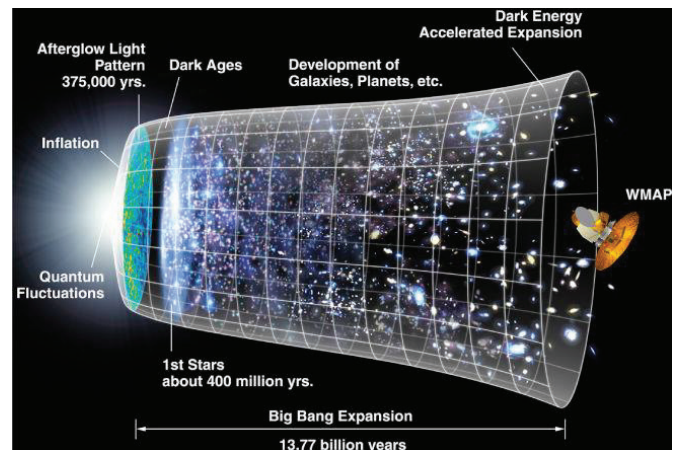


Figure 1: The figure shows the evolution of the universe from early inflation and structure formation to a late-time, dark energy-dominated phase characterized by accelerated expansion. Credits - Wilkinson Microwave Anisotropy Probe (WMAP) and National Aeronautics and Space Administration (NASA).

Section 2 - Dark Energy as the Cosmological Constant in the Λ CDM Model:

The Λ CDM model, which is the most widely accepted theory, provides the simplest explanation for the acceleration of

the cosmic expansion. Λ CDM stands for Lambda Cold Dark Matter. It combines dark energy with cold dark matter and general relativity.⁴ It explains several observations in cosmology, like baryon acoustic oscillations (BAO), observations from Type Ia supernovae, and anisotropies of the Cosmic Microwave Background (CMB). The regular periodic fluctuations in the density of ordinary matter that are visible in the universe are defined as baryon acoustic oscillations. They often help in determining distances on a cosmic scale.¹⁵ Observations from Type Ia supernovae are analysed for measuring the acceleration of the expansion rate of the universe. The small difference in the temperature of various regions of the CMB is referred to as the anisotropies of the CMB. These differences were measured by missions such as Planck and WMAP.⁴ The Λ CDM model has achieved a high degree of accuracy with respect to the observations from large-scale structures by integrating all these observations simultaneously. Dark energy serves as the cosmological constant Λ in this model. Although this theory agrees with observational data, it has major theoretical issues like the cosmological constant problem, Hubble Tension, and the energy conservation paradox.

The cosmological constant Λ was introduced in 1917 by Albert Einstein in his equations of general relativity. During his time, it was believed that the universe was static and was not expanding. In contrast, his equations initially implied that the universe was either expanding or contracting. For the equations to predict a stationary universe, he included Λ as a repulsive force that balanced gravity.⁶ However, this term was theoretical and had no observational evidence to prove its existence.

In 1929, Edwin Hubble discovered that the universe was indeed expanding. Consequently, Einstein said that including Λ in his equations was his "greatest blunder". Λ was considered unnecessary in cosmology for several decades. But this changed in the late 1990s. Two independent teams analysing observations from Type Ia supernovae found that the expansion rate of the universe was accelerating.⁸ This discovery made Λ relevant again, and it was now considered a constant form of energy that pushes the universe apart.

It was predicted in the Quantum Field Theory that the vacuum energy in space should have an enormous energy density. This value is 10^{120} times greater than the value that has been obtained from observations. This is known as the cosmological constant problem. A large amount of fine-tuning helps to reduce the magnitude of this discrepancy, which is considered to be unsatisfactory.¹⁶

In classical Newtonian physics, the energy of a closed system is always conserved. But the energy density of dark energy in the universe is a constant and does not dilute with expansion. This is not because energy is physically created, but because dark energy remains uniform as it is an inherent property and is hence not affected by the expansion of the universe. This raises questions about the applicability of the traditional energy conservation principle. Energy conservation in a global context is still not well defined in general relativity. This is a major roadblock to scientists who wish to formulate a perfect theory of everything.¹⁴

The difference between the value of the expansion rate of the universe (H_0) measured using Cepheid variables and Type Ia supernovae and the value obtained from the CMB is referred to as the Hubble Tension.¹⁷ The value of H_0 obtained from the distance ladder methods is 73 km/s/Mpc, while the value of H_0 obtained from CMB observations is 67 km/s/Mpc. Currently, the error bar in the value obtained from the distance ladder method is ± 1.04 , while the error bar in the value obtained from CMB observations is ± 0.80 . As the error in measurement is decreased, the difference between the two values also increases. This indicates a need to revise our current cosmological model.¹⁷

The Λ CDM model is still the most widely accepted because of its simple framework and its accuracy in regard to observational data. According to the Λ CDM model, dark energy is constant, dark matter is cold and non-relativistic, while the rest is ordinary matter and radiation. It is often used as a baseline against which theories that are more complex and speculative can be compared because of its clarity. The Λ CDM model is also very successful in explaining the expansion of the universe. As a result, it is still used for large-scale surveys and simulations in cosmology. It is the closest approximation to the universe's history of events that we currently have. However, in the future, it should be replaced by a model that solves all of these flaws while still being accurate in regard to observational data.⁴ Therefore, by modifying the Λ CDM model by utilizing other theories such as Noether's Theorem, we can get one step closer to solving a paradox that has been confusing scientists for years.

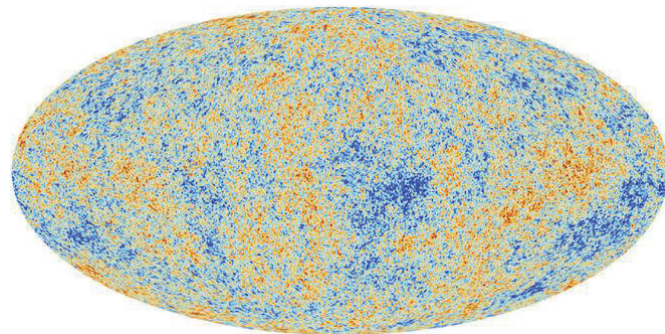


Figure 2: The figure shows the 2013 Planck all-sky map of the Cosmic Microwave Background temperature is shown. The small difference in the temperature of various regions of the CMB, referred to as anisotropies, provides strong observational evidence that the universe is spatially flat and supports the Λ CDM model, in which dark energy dominates the universe's late-time expansion. Credits - European Space Agency (ESA) and the Planck Collaboration.

Section 3 - Quantum and relativistic approaches:

This section examines quantum and relativistic approaches to explain dark energy and its constant density.

3.1. - Quantum Field Theory:

Quantum Field Theory has been used to try to explain this paradox. Vacuum energy is another term used instead of dark energy. The vacuum of space is filled with fluctuating energy in quantum field theory. The vacuum (dark) energy density predicted by quantum field theory is 10^{120} times greater than the observed values. This gives rise to the cosmological

constant problem.¹⁶ This is one of the most famous inconsistencies between theory and observations in astrophysics. This is considered the cosmological constant problem since this huge mismatch highlights the discrepancy between QFT and our observations. The magnitude of the difference in the values can be decreased by using renormalisation techniques. However, it cannot be completely removed. This is still a major problem that hinders our understanding of dark energy in the expanding universe.

3.2. - *Semiclassical Gravity:*

Semiclassical gravity is one method that can be used to connect quantum mechanics and gravity. In semiclassical gravity, quantum fields propagate on a classical spacetime background.¹⁸ Vacuum expectation values can be calculated using the framework. But it fails to consistently include backreaction, which becomes a problem when we consider an evolving universe. The phenomenon of backreaction is explained further in the next section. Also, when the quantum fluctuations become too large or when non-classical states are involved, this method can no longer be applied.

3.3. - *Modified Gravity Theory:*

Modified gravity theories can also be used, which adjust Einstein's equations without adding dark energy as Λ . $f(R)$ gravity and scale-tensor models attempt to explain accelerating expansion without using the cosmological constant Λ (dark energy).¹⁹ Corrections based on curvature are introduced to the action of gravity in these theories, which can result in an acceleration in the expansion rate of the universe. However, for $f(R)$ gravity models to be viable, they need to satisfy several conditions, such as avoiding instability and passing local gravity tests. Hence, building models utilising these theories becomes complicated.

3.4. - *Other Mathematical Frameworks and Limitations:*

Mathematical frameworks such as Lagrangian and Hamiltonian mechanics can define conserved quantities in stationary (static) spacetimes. However, they cannot clearly define these quantities in an expanding universe.²⁰ The absence of time translational symmetry in an expanding universe in a global context means that energy conservation cannot be directly applied.²¹

There is no quantum or relativistic approach that currently clearly explains the reason for the creation of energy as the universe expands without violating conventional energy conservation. Each theory has its respective flaws. The estimated values from Quantum Field Theory are much greater than the values obtained from observations, while semiclassical gravity models are inconsistent. Modified gravity theories have several flaws that require fine-tuning, while theories using mathematical frameworks such as Hamiltonian mechanics are not applicable in evolving spacetimes. None of these frameworks perfectly solves this paradox; however, they bring us closer to finding this solution. The models obtained from quantum and relativistic approaches have several flaws and also require ex-

cessive assumptions. What if we just discard the cosmological constant?

Section 4 - Backreaction from Cosmic Structure Formation:

This section talks about the backreaction hypothesis, which states that the accelerating cosmic expansion could be due to cosmic backreaction rather than dark energy. This removes the requirement for dark energy in cosmology and its associated problems and challenges.

The universe is assumed to be homogeneous and isotropic on large scales. However, it contains smaller, complex structures such as galaxies and voids, which evolve nonlinearly.²² The universe's expansion rate can be changed because of the uneven growth of the cosmic web, which is formed by these structures. These small-scale inhomogeneities are not smoothed over the fabric of the universe. According to Einstein's field equations, these inhomogeneities can change the average expansion rate and change the evolution path of the universe from that of an ideal FLRW universe. An ideal FLRW universe is a homogeneous, isotropic, and expanding universe that is path-connected. Therefore, it is not possible to include the dynamics of the universe using a perfectly smooth model.²³

Λ is interpreted as the energy density of dark (vacuum) energy in modern cosmology and accounts for the accelerating expansion of the universe. But the backreaction hypothesis tries to explain this phenomenon without utilising Λ . The emphasis is thereby shifted to the distribution of matter and the geometry of spacetime. By rendering Λ unnecessary, this hypothesis proposes a new way to integrate acceleration of the expansion rate with general relativity by ignoring one of the most fundamental assumptions in modern cosmology.

Backreaction is a geometrical effect that occurs because of the averaging of expansion rates across the universe. The expansion of regions with lower density is faster than the collapse of regions with higher density. As a result, the average expansion rate can increase even though no local region is accelerating because of the average combined effect.²⁴ Even if the expansion of a local region of space is not accelerating, the average expansion rate might still accelerate. The effects of the lower-density regions that have a higher growth rate can overshadow the effects of the higher-density regions that have a lower growth rate. This effect mimics that of a positive cosmological constant.²⁵

The Buchert formalism helps us to average Einstein's Field Equations over the regions in the universe that have an uneven distribution of matter. It also gives us additional terms that can affect the expansion of the universe.²⁴ If these terms grow much larger over time, they can produce an acceleration in the average expansion rate of the universe without needing a component that has negative pressure. Theoretically, this hypothesis is viable as an alternative to the Λ CDM model.

The Λ CDM model can explain the observed supernovae data without using the cosmological constant. However, this hypothesis requires additional assumptions to explain all the cosmological data, such as BAO and CMB anisotropies. Furthermore, for the universe after recombination to be consistent with CMB data, it has to be extremely close to being

perfectly homogeneous and isotropic. As a result, only a limited amount of inhomogeneity can develop in the universe. At high redshifts, the backreaction hypothesis predicts a different expansion history than the one in the Λ CDM model, which is different from the data provided by various probes.²⁶

If the accelerating expansion of the universe is caused by backreaction, there is no longer a need for dark energy. As a result, there would no longer be any challenges associated with energy conservation due to dark energy. If acceleration is caused only by the matter distribution and geometry of the universe, there would no longer be a violation. Local energy conservation would still apply regionally, but the average expansion rate would still accelerate due to backreaction. In this hypothesis, Noether's theorem would not play any role. This is because time translation symmetry in such a universe would be broken by inhomogeneity on a large scale.²⁷ As a result, the conservation principles would depend on geometry and would only work locally. This would offer an extremely different interpretation of the dynamics of the universe. The concept of time translation symmetry is explained in the next section.

Section 5 - Noether's Theorem and Its Implications:

5.1. - History of Noether's Theorem:

Instead of indicating an outright violation of the conservation of energy, dark energy's constant density may be explained by a different symmetry in the evolving universe.²⁸ Noether's Theorem provides a simple explanation for the constant density of dark energy by connecting symmetry to the conservation laws. Emmy Noether's theorem was formulated in 1915 and published in 1918.²⁹ Now it has become an integral part of modern theoretical physics. It talks about the relation between the various symmetries of a system and the associated conserved quantities in those systems. In essence, if the laws of the system remain unchanged even after certain transformations, a need for a conserved quantity arises. For example, the unchanging symmetry of physical systems with respect to time gives way to the energy conservation principle. This helps us to explain why some properties do not change with the evolution of a system. Another idea that can be inferred from the law is that if a conserved quantity is present, it refers to the presence of a specific law of a system that remains unchanged even after transformation.²⁹ Mathematically, Noether's Theorem states that for every continuous symmetry of the action S , there exists a conserved current J^μ .³⁰

$$\partial_\mu J^\mu = 0 \quad (2)$$

This equation means that the divergence of the current J^μ disappears, so the total amount of the associated conserved quantity remains constant over time when integrated across space. Here, ∂_μ represents the derivative with respect to space-time coordinates, and J^μ is the conserved current related to a symmetry of the system. In the case of time-translation symmetry, the conserved quantity is energy. This means that the integral of the conserved quantity over space remains constant in time, which is the principle behind local energy conservation according to Noether's Theorem.³⁰

5.2. - Noether's Theorem and the Λ CDM model:

When Noether's theorem is applied to our current model of the universe, the Λ CDM model, in general relativity, the flat but dynamic nature of spacetime complicates the usual time translational symmetry. This is because the universe is expanding, which affects the concerned symmetries. As a result, the idea of globally conserved energy becomes complicated. Consequently, the regular time translational symmetry used along with energy conservation cannot be directly applied. Hence, the idea that our current energy conservation laws work on a global scale is challenged.¹⁴ Regardless, local energy conservation is still valid as the covariant divergence of the energy-momentum tensor disappears. This is because time translational symmetry is present in an unchanging state, which allows for the definition of a conserved quantity.³¹

Noether's Theorem works best in flat universes that are static. This is because time translational symmetry is enough to reconcile the constant dark energy density with energy conservation.³² It provides a mathematical framework that contains formulas that permit dark energy to be constant without violating energy conservation. Therefore, energy conservation needs to be approached in a novel manner by considering the symmetries that affect spacetime.²¹ Hence, dark energy should be viewed as a natural consequence of the structure of spacetime rather than an anomaly.³² Noether's Theorem also works in flat universes that are expanding, but to a more restricted extent.

A modified conservation law compatible with the constant density could come into existence if the expansion of space-time itself encodes a symmetry.³² As of now, no symmetry directly solves this paradox with respect to expanding curved spacetimes. This is because the symmetries of curved spacetimes differ from those in flat ones, since the spacetime undergoes evolution and gets stretched. However, with further research, new symmetries can be discovered that help to reconcile this constant density with the broader energy conservation principle, as these evolving symmetries can help in the adaptation of conservation laws in an expanding universe.

Accordingly, the problem concerning the constant density of dark energy gives rise to a need for the revision of the traditional form of energy conservation. It should be replaced with conservation laws that are based on symmetry and are hence suitable for an evolving universe. If we can apply Noether's Theorem to curved spacetimes, we would be able to resolve this paradox of constant energy density in the most general case very easily.³⁴ As a result, the need for more assumptions, such as those needed in Quantum Field Theory and modified gravity theories, is avoided, and the question now focuses more on finding the correct spacetime symmetry.

5.3. - Advantages of Noether's Theorem:

Unlike in Quantum Field Theory and modified gravity, Noether's theorem has minimal assumptions in its framework.³¹ It is assumed that symmetries are continuous, transformations are well defined, and the boundary conditions vanish. Other models that have been used to explain the constant dark energy density often utilize complex physics and assumptions.

However, Noether's Theorem offers a model that explains this in a simpler and mathematical way.⁹ By applying Noether's Theorem to our current cosmological model, astrophysicists can strive to obtain a new modified conservation law that is in agreement with the observed constant density of dark energy in the expanding universe. The approach using Noether's Theorem suggests that the violation of conventional energy conservation exists because we apply traditional definitions without taking into consideration the change in spacetime symmetries, rather than considering it as a direct violation of the energy conservation principle.

■ Conclusion

A significant challenge to the principle of energy conservation on large scales is presented by the assumption of constant density of dark energy in an expanding universe. This review goes over a few theoretical attempts that try to solve this paradox, from general relativity and quantum field theory, to large-scale backreaction and symmetry. Some further questions that may arise from this paper are:

a) Could there be a theory of quantum mechanics that completely integrates cosmic expansion and its effects with conservation laws? b) What further conditions are required for backreaction to completely replace dark energy as the driving force for accelerating expansion of the universe in cosmology?

By reviewing quantum and relativistic models and the effects of cosmic backreaction, it becomes obvious that this paradox is still unsolved in many popular approaches. Although Noether's Theorem is not accepted as the final answer, it provides a mathematically valid framework that connects modified forms of conservation principles to evolving cosmological symmetries. However, alternative models and theories also offer compelling ideas that do not violate the conventional principle of energy conservation. Further research is essential to solve this paradox. Solving this paradox would help us to properly incorporate quantum theory into our current model of cosmology. Furthermore, it could help us to ultimately understand the fundamental forces in our universe by developing a true theory of everything backed by quantum mechanics.

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■ References

- Perlmutter, Saul, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, *et al.* 1999. "Measurements of Ω and Λ from 42 High-Redshift Supernovae." *The Astrophysical Journal* 517 (2): 565–86. <https://iopscience.iop.org/article/10.1086/307221/pdf>
- Mantz, Adam, Steven W. Allen, and David Rapetti. 2008. "New Constraints on Dark Energy from the Observed Growth of the Most X-Ray Luminous Galaxy Clusters." *Monthly Notices of the Royal Astronomical Society* 387 (3): 1179–88. <https://www.slac.stanford.edu/pubs/slacpubs/12750/slac-pub-12852.pdf>
- Hoekstra, Henk, and Bhuvnesh Jain. 2008. "Weak Gravitational Lensing and Its Cosmological Applications." *Annual Review of Nuclear and Particle Science* 58 (1): 99–123. <https://arxiv.org/pdf/0805.0139>
- Planck Collaboration. 2021. "Planck 2018 Results. VI. Cosmological Parameters." *Astronomy & Astrophysics* 641: A6. <https://arxiv.org/abs/1807.06209>
- O'Raifeartaigh, Cormac, Michael O'Keeffe, Werner Nahm, and Simon Mitton. 2018. "One Hundred Years of the Cosmological Constant: From 'Superfluous Stunt' to Dark Energy." *The European Physical Journal H* 43 (1): 73–117. <https://arxiv.org/pdf/1711.06890>
- Weinberg, Steven. 1989. "The Cosmological Constant Problem." *Reviews of Modern Physics* 61 (1): 1–23. <https://profchristophberger.com/wp-content/uploads/2015/02/wei89.pdf>
- Harko, Tiberiu, Francisco S. N. Lobo, G. Otafora, and Emmanuel N. Saridakis. 2014. "f(T, T) Gravity and Cosmology." *arXiv preprint arXiv:1405.0519*. <https://doi.org/10.1088/1475-7516/2014/12/021>
- Riess, Adam G., Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, *et al.* 1998. "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant." *The Astronomical Journal* 116 (3): 1009–38. <https://doi.org/10.1086/300499>
- Padmanabhan, T. 2008. *Dark Energy and Gravity*. *arXiv preprint arXiv:0802.1798*. <https://arxiv.org/pdf/0705.2533>
- Ellis, George F. R. 2006. "Physics in the Real Universe: Time and Spacetime." *arXiv preprint arXiv:gr-qc/0605049*. <https://arxiv.org/pdf/gr-qc/0605049>
- Wu, Zhaoyan. 2007. *No-Go Theorem for Energy-Momentum Conservation in Curved Spacetime*. *arXiv preprint arXiv:gr-qc/0702059*. <https://arxiv.org/pdf/gr-qc/0702059v1>
- Pachner, Jaroslav. 1965. "Problem of Energy in an Expanding Universe." *Physical Review* 137 (5B): B1379. <https://doi.org/10.1103/PhysRev.137.B1379>
- Peebles, P. J. E., and Bharat Ratra. 2003. "The Cosmological Constant and Dark Energy." *Reviews of Modern Physics* 75 (2): 559–606. <https://arxiv.org/pdf/astro-ph/0207347>
- Carroll, Sean M. 2001. *Dark Energy and the Preposterous Universe*. *arXiv preprint arXiv:astro-ph/0107571*. <https://arxiv.org/abs/astro-ph/0107571>
- Eisenstein, Daniel J., Idit Zehavi, David W. Hogg, Roman Scocimarro, Michael R. Blanton, Robert C. Nichol, Ryan Scranton, *et al.* 2005. "Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies." *arXiv preprint arXiv:astro-ph/0501171*. <https://arxiv.org/abs/astro-ph/0501171>
- Martin, Jérôme. 2012. "Everything You Always Wanted to Know About the Cosmological Constant Problem (But Were Afraid to Ask)." <https://arxiv.org/pdf/1205.3365v1>
- Scolnic, Dan, Dillon Brout, Gautham Narayan, Adam G. Riess, Dragan Huterer, Jessica E. Yoachim, Armin Rest, *et al.* 2024. "The Hubble Tension in Our Own Backyard: DESI and the Nearness of the Coma Cluster." *The Astrophysical Journal Letters* 974 (1): L12. <https://arxiv.org/pdf/2409.14546>
- Ford, L. H. 2005. "Spacetime in Semiclassical Gravity." *arXiv preprint arXiv:gr-qc/0504096*. <https://arxiv.org/pdf/gr-qc/0504096>
- Sotiriou, Thomas P., and Valerio Faraoni. 2010. "f(R) Theories of Gravity." *Reviews of Modern Physics* 82 (1): 451–97. <https://arxiv.org/pdf/0805.1726>
- Capozziello, Salvatore, and Shinji Tsujikawa. 2008. "Solar System and Equivalence Principle Constraints on f(R) Gravity by the Cha-

- meleon Approach." *Physical Review D* 77 (10): 107501. <https://arxiv.org/pdf/0712.2268>
21. Brown, J. David. 1993. "Action Functionals for Relativistic Perfect Fluids." *Classical and Quantum Gravity* 10 (8): 1579–1606. <https://arxiv.org/pdf/gr-qc/9304026>
 22. Clarkson, Chris, George Ellis, Julien Larena, and Obinna Umeh. 2011. "Does the Growth of Structure Affect Our Dynamical Models of the Universe? The Averaging, Backreaction, and Fitting Problems in Cosmology." *Reports on Progress in Physics* 74 (11): 112901. <https://arxiv.org/pdf/1109.2314>
 23. Buchert, Thomas. 2000. "On Average Properties of Inhomogeneous Fluids in General Relativity: Dust Cosmologies." *General Relativity and Gravitation* 32: 105–25. <https://arxiv.org/pdf/gr-qc/9906015>
 24. Kolb, Edward W., Sabino Matarrese, Alessio Notari, and Antonio Riotto. 2005. "Effect of Inhomogeneities on the Expansion Rate of the Universe." <https://arxiv.org/pdf/hep-ph/0409038>
 25. Kolb, Edward W., Sabino Matarrese, and Antonio Riotto. 2006. "On Cosmic Acceleration without Dark Energy." *New Journal of Physics* 8 (12): 322. <https://iopscience.iop.org/article/10.1088/1367-2630/8/12/322/pdf>
 26. Bolejko, Krzysztof, Andrzej Krasinski, Charles Hellaby, and Marie-Noëlle Célérier. 2011. "Inhomogeneous Cosmological Models: Exact Solutions and Their Applications." *Classical and Quantum Gravity* 28 (16): 164002. <https://arxiv.org/pdf/1102.1449>
 27. Buchert, Thomas, Alan A. Coley, Hagen Kleinert, Boudewijn F. Roukema, and David L. Wiltshire. 2015. "Is There Proof That Backreaction of Inhomogeneities Is Irrelevant in Cosmology?" *Classical and Quantum Gravity* 32 (21): 215021. <https://arxiv.org/pdf/1505.07800>
 28. Paliathanasis, Andronikos, Michael Tsamparlis, Salvatore Basilakos, and Spyros Capozziello. 2011. "Constraints and Analytical Solutions of $f(R)$ Theories of Gravity Using Noether Symmetries." *Physical Review D* 89 (6): 063532. <https://arxiv.org/pdf/1111.4547>
 29. Noether, Emmy. 1971. "Invariant Variation Problems." Translated by M. A. Tavel. *Transport Theory and Statistical Physics* 1 (3): 186–207. <https://arxiv.org/abs/physics/0503066>
 30. Barnich, Glenn, and Friedemann Brandt. 2002. "Covariant Theory of Asymptotic Symmetries, Conservation Laws and Central Charges." *Nuclear Physics B* 633 (1–2): 3–82. <https://arxiv.org/pdf/hep-th/0111246>
 31. Capozziello, Salvatore e Antonio De Felice. 2008. " $f(R)$ Cosmology by Noether's Symmetry." *Journal of Cosmology and Astroparticle Physics* 2008 (08): 016. <https://arxiv.org/pdf/0804.2163>
 32. Vakili, Babak. 2008. "Noether Symmetry in $f(R)$ Cosmology." *Physical Review D* 77: 044023. <https://arxiv.org/pdf/0804.3449v1>
 33. Tsamparlis, Michael, and Andronikos Paliathanasis. 2018. "Symmetries of Differential Equations in Cosmology." *Symmetry* 10 (7): 233. <https://arxiv.org/pdf/1806.05888>
 34. Brown, J. David, and James W. York. 1993. "Quasilocal Energy and Conserved Charges Derived from the Gravitational Action." *Physical Review D* 47 (4): 1407–19. https://web.archive.org/web/20180728121612id_/http://cds.cern.ch/record/244126/files/9209012.pdf

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