

Virtual Power Plant Incentive Strategy Based on the SIR Model and Evolutionary Game Theory

Ke Liu

Valor Christian High School, 3775 Grace Blvd, Highlands Ranch, CO, 80126, USA; ke862609@gmail.com
Mentor: Huaqian Xiao

ABSTRACT: As the global industries' need for new energy increases daily, more electrical engineers emphasize the benefits of the Virtual Power Plant (VPP), a digital coordination of distributed energy resources. By rationally adjusting electricity incentives and staggering electricity usage, the Virtual Power Plant reduces the power grid's burden and lowers the electricity price, maintaining power grid stability and benefiting collaborators. Such a positive effect is directly related to the size of VPP's collaborator base. This paper proposed a method to optimize the collaborator base while maximizing the profit of VPP through electricity incentives. The method originates from basic SIR models, integrates revolutionary game theory to reflect the variety of collaborators, and develops a bilevel model to illustrate the dynamic relationship among the incentive, the collaborator base, and VPP's profit. The paper also includes a case study of a local Chinese electricity department's VPP operation to demonstrate the superiority of this method in guiding VPP's incentives. The study shows that the proposed method can effectively increase the cooperation rate to 85% and the profit by 16.3% compared to before using the method. Such a result proves the method's reliability and effectiveness in maximizing VPP's profit.

KEYWORDS: Energy: Physical, Electricity, Virtual Power Plant, Incentives, SIR Model.

■ Introduction

Virtual Power Plant (VPP), a digital network of distributed energy sources, is becoming more pervasive around the world as the demand for clean energy is increasing. Normal VPPs have a centralized control structure, as shown in Figure 1, where individual energy resources and electricity consumers are directly connected to control coordination centers.¹ The coordination center serves to efficiently use energy, reduce burden on the energy grid, and ensure energy usage's stability and security by rationally allocating power supply for consumption and encouraging staggered electricity use.¹

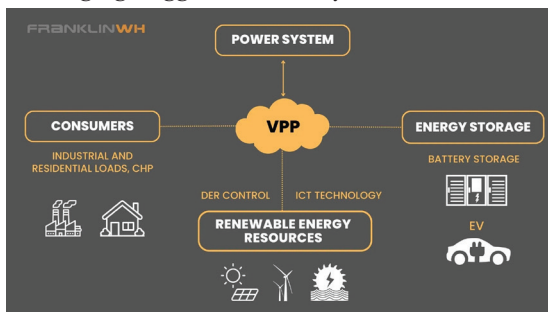


Figure 1: Virtual power plant diagram.² The centralized structure of VPP is shown by connecting the consumers and three different types of producers.

The effective allocation of energy resources enables VPPs to reduce power supply costs, enabling them to lower electricity prices below market levels to attract more consumers. Therefore, the proper functioning of VPP creates a positive spiral in which a larger consumer base gives VPP greater control over the energy grid, and greater control leads to more efficient energy prices. Commercial VPP profits in two ways: energy sales and government incentives.³ Governments provide incentives

to promote the adoption and integration of renewable energy.³ Again, a larger consumer base leads to better cooperation of the VPP; therefore, the expansion of collaboration is the essence of both the VPP and the consumer's profit.

One distinct way to foster collaboration is VPP's incentive to the collaborating enterprises.³ Much previous research has studied methods to analyze the economic relationship between VPP's pricing and market price.³ By using cooperative game theory, Yafei Wang, Weijun Gao, Fanyue Qian, and You Li analyze how to rationally allocate the profit from VPP operation to the demand and plant side, in forms such as incentives and tariffs, to ensure all participants are incentivized to join and maintain the long-term collaboration.⁴ Additionally, Zhengyu Shu and colleagues apply the Stackelberg game to create a bi-level model that demonstrates the market's reaction to VPP's pricing to satisfy demand and the effect of the reactions on VPP's final profit.⁵ However, current studies on consumers' reactions to VPP's incentive price often simplify consumers as homogenous individuals, overlooking the differences between them and their influence on each other's decision-making. Instead of assuming that consumers are completely and ideally rational when responding to incentive prices, research should analyze how collaboration can dynamically spread and collapse within a heterogeneous consumer base.⁶

Therefore, the researchers adopt SIR (Susceptible-Infected-Recovered) models as a foundation to simulate the dynamic changes, fueled by incentives, in collaborative relationships between consumer groups and the VPP. The SIR model is originally used in biomedicine to predict the spread of certain diseases. This article specifically uses the SIRS model, which contains the additional reinfection rate and is used

for diseases that only create temporary immunity.⁷ In the VPP scenario, the disease is the collaboration of consumers; the susceptible population is the potential collaborators; the infected population is the current collaborators; and the recovered population is the cooperation withdrawers. Therefore, integrating the SIR model can vividly illustrate the state of collaboration within the consumer base. However, the basic SIR model applies only to a homogeneous population, assuming constant infection, recovery, and reinfection rates.⁷ The researchers then develop a system for categorizing consumers and create dynamic, averaged rates by integrating evolutionary game theory, prospect theory, and the Capital Asset Pricing Model into the definition of the rates. Furthermore, researchers utilize a bilevel model to connect such dynamic representations of collaboration to the total profit of the VPP, creating a timely, effective guideline on the VPP's incentive pricing to maximize its profit; in this way, the proposed model solves the problem and uncertainty of VPP incentive pricing in the complicated, heterogeneous market and facilitates energy companies to more likely achieve their targeted profit.

■ Methods

Materials

The conducted study uses no physical materials. Instead, the main materials only include key academic theories and Google Colab for the model solving. The specific consumers' information in the case study is obtained from a virtual power plant operator in a city in western China.

Model Derivation:

- Reflection of consumers' heterogeneity and model inputs setup:

In order to precisely represent the heterogeneity of the consumer population, the researchers first categorize the consumers into large, medium, and small according to their electricity consumption.

In the case of the virtual power plant operator in a city in western China, an enterprise with 0-1 MWh electricity usage is considered small, an enterprise with 1-5 MWh is considered medium, and an enterprise with greater than 5 MWh usage is large.

This criterion divides the total consumers into three groups evenly, meaning each group has a similar and fair impact on the average values. In different cases, the specific criterion depends on the specific companies and the distribution of different usage quantities on the scale.

Additionally, the consumers can be categorized into different sensitive levels to the electricity price, R_i , according to the following equation:

$$R_i = \text{annual electricity expenditure} / \text{annual revenue} * 100\%$$

Highly sensitive enterprises have $R_i \geq 4\%$, medium sensitive enterprises have $1\% \leq R_i \leq 4\%$, and low sensitive enterprises have $R_i < 1\%$

Based on the sizes of different consumers and different sensitivities, the researchers developed three other variables, k_i , $risk_i$, and σ_i to reflect the heterogeneity's change to the original rates of the SIR model. k_i reflects the rationality of consumer

i , measuring how readily the consumer's decision is affected by the social environment. Xianjia Wang, Culing Gu, Jinhua Zhao, and Ji Quan use the Fermi function to explain the impact of k on an individual's decision-making: when $0 \leq k < 1$, the individual is perfectly rational; as the k value increases till 20, the individual tends to rely more on their environment.⁸ Therefore, k base equals 8, implying typical human behavior - neutral rationality - in decision-making. Then, k is multiplied by different coefficients to represent the behavioral tendency according to the sensitivity of different consumers. $risk_i$ is the risk reversion of consumers, in other words, how likely they are to end their collaboration with the VPP due to potential risk. The risk base is set to 1, referencing the beta, which measures investment volatility in financial economics, a beta of 1 serves as a benchmark for enterprises that vary exactly with market swings. A beta greater than 1 indicates enterprises that are more volatile than the market, while a beta less than 1 indicates less volatile ones. Therefore, in the case of virtual power plants, the risk base is multiplied by different coefficients based on size, making it equal to, less than, or greater than 1: large consumers are willing and able to take losses, while smaller consumers are more sensitive. Larger consumers have a smaller $risk_i$, meaning they are less responsive and volatile to market risks, while smaller consumers are the opposite. σ_i models consumers' decisions' impact on society. In order to represent different levels of impact by different sizes of consumers, a σ base is needed. Tverskoi, Badu, and Gavrilits analyze the significant impact of the public's acceptance of a technology on the spreading of such technology, and model conformity of peers, which is a key factor in an individual's acceptance, into an S-shaped diffusion curve.⁹ According to Tverskoi and his peers, the S-shaped diffusion curve is commonly shown when modeling problems that involve the spread of social behaviors. For a clear and realistic S-shaped diffusion curve of collaboration, the base value is set to 1, multiplied by different coefficients based on the consumer's size: larger enterprises' decision-making processes are usually private and independent, whereas small enterprises' results are more easily observed and mimicked by other enterprises in society. All the multipliers used in this section are illustrative, but reasonable parameters to quantify and simulate the heterogeneity among consumers. Through algorithm experimentation, the model's results are stable and reliable within the range of $\pm 15\%$ to $\pm 35\%$ of the illustrative multipliers. Based on their size and sensitivity, enterprises are divided into 9 categories, summarized in Table 1.

Table 1: Categories of enterprises. Categorize diverse consumers into 9 groups with corresponding rationality, risk aversion, and impact level based on size and sensitivity to price change.

Electricity Sensitivity	Size	Rationality k_i	Risk Aversion $risk_i$	Impact on society σ_i
High	Large	8*1.25	1*0.8	1*0.7
High	Medium	8*1.25	1*1.0	1*1.0
High	Small	8*1.25	1*1.3	1*1.3
Medium	Large	8*1.0	1*0.8	1*0.7
Medium	Medium	8*1.0	1*1.0	1*1.0
Medium	Small	8*1.0	1*1.3	1*1.3
Low	Large	8*0.75	1*0.8	1*0.7
Low	Medium	8*0.75	1*1.0	1*1.0
Low	Small	8*0.75	1*1.3	1*1.3

- Development of the dynamic SIR model:

The Basic SIR model is the following:

$$\begin{aligned}\frac{dS}{dt} &= -\beta(m) \cdot I(m) \cdot \frac{S(m)}{N} + \delta(m) \cdot R(m) \\ \frac{dI}{dt} &= \beta(m) \cdot I(m) \cdot \frac{S(m)}{N} + \gamma(m) \cdot I(m) \\ \frac{dR}{dt} &= \gamma(m) \cdot I(m) - \delta(m) \cdot R(m)\end{aligned}$$

Specifically, the infection rate, $\beta(m)$, is the monthly rate that potential collaborators become real collaborators; the recovery, $\gamma(m)$, is the monthly rate that collaborators exit collaboration; and the reinfection rate, $\delta(m)$, is the monthly rate that recovered enterprise reenters collaboration. N , $S(m)$, $I(m)$, and $R(m)$ are the monthly total population, susceptible population, infected population, and recovered population. Below are the specific steps to change those constant rates into dynamic rates to better illustrate the inter-enterprise influence and real-time changes in corporate decisions.

a. Dynamic $\beta(m)$:

Researchers utilize the Fermi Function to illustrate how an enterprise's decision to join or not depends on its perception of profit difference. The Fermi Function is a common rule used in Evolutionary Game Theory, a model of individuals interacting through strategic games, where success depends on how widespread their strategies are.¹⁰ The advantage of such game theory is that it allows individuals to adjust their strategies in response to environmental changes; in other words, it does not assume that all individuals are rational.⁸ In the case of VPP, the strategies discussed are whether to join the collaboration, where joined enterprises can influence others' strategies and turn them into collaboration. Specifically, the Fermi Function explains the linear relationship between profit difference and the probability of individuals taking certain strategies.⁸ Therefore, the Fermi Function is a great tool to define $\beta(m)$, the spread of collaboration. The basic formula of the Fermi Function is below:¹⁰

$$p_f(\pi, \pi') = \frac{1}{(1 + e^{-s(\pi' - \pi)})}$$

$p_f(\pi, \pi')$ is the probability for individuals to change their decision from π to π' based on the profit difference $(\pi' - \pi)$ and weights.

Therefore, the probability for individual consumer i to change from non-cooperation to cooperation can be written as:

$$G_i(\Delta\pi_i(m)) = \frac{1}{1 + e^{-k_i \cdot \Delta\pi_i(m)}}$$

k_i is the rationality of consumer i , meaning how easily influenced the consumer is by changes in incentives. $\Delta\pi_i$ is the profit difference, defined $incentive_j(m) \cdot p_i - c_i^{adj}$, where $incentive_j(m)$ is the incentive provided that month, p_i is the adjustable power, and c_i^{adj} is the collaboration cost.

Then, calculate the weight average of everyone's collaboration rate to obtain $\beta(m)$:

$$\beta(m) = \frac{(\sum_{i=1}^N [w_i \cdot G_i(\Delta\pi_i(m))])}{N}$$

$w_i = \frac{(p_i \cdot \sigma_i)}{\sum_{i=1}^N (p_i \cdot \sigma_i)}$, representing the level of impact of consumer i 's decision on society.

b. Dynamic $\gamma(m)$

The basic definition of $\gamma(m)$ can be written as:

$$\gamma(m) = \gamma_{0,i} \cdot \eta_{risk,i} \cdot \eta_{profit,i}(m)$$

$\gamma_{0,i}$ is the basic exit rate that occurs in human natural tendency: according to Reinartz and Kumar, businesses cannot decrease their cooperation exit rate to zero simply by adjusting operation strategies; therefore, there is a natural, inherent exit rate.¹¹ The exact value of $\gamma_{0,i}$ which is 0.02/month, is calculated from enterprises' account retention over years, including Ameritrade and Capital One, in the study by Sunil Gupta, Donald Lehmann, and Jennifer Stuart, and verified by the customer turnover of the cellular telephone industry, which 2.7% per month, in the study by Roth Bolton.^{12,13} $\eta_{risk,i}$ is the risk factor, representing consumers' exit from collaboration due to risk aversion. $\eta_{profit,i}$ is the profit factor, representing consumers' exit from collaboration due to the failure to realize their ideal profit.

The definition of profit factor is based on the prospect theory, which is shown below:

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]$$

, which includes choice x and y and their outcome, respectively, p and q . $\pi(p)$ is the weight, showing how sensitive individuals are to failure in profit. $v(x) - v(y)$ is the comparison between expected profit and actual profit.

Therefore, in the VPP scenario, the prospect theory can be applied as follows:

$$\eta_{profit,i}(m) = 1 + \alpha \cdot \max(0, \Delta\pi_{threshold} - \Delta\pi_i(m))$$

$\Delta\pi_{threshold}$ is how much profit the consumer i expects to gain by collaborating, and is set to be $1.2^* c_i^{adj}$ because consumers expect to obtain basic profit outside of the cost; $\Delta\pi_i(m)$ is the same as in section a. α is set to be 0.5 to illustrate a clear SIR curve, avoiding excessively high attrition rates leading to the collapse of the collaborative group.

The definition of the risk factor is based on the linear relationship between the consumers' expected profit and the level of risk, shown by the Capital Asset Pricing Model. Therefore, the risk level has a linear effect on the $\gamma(m)$:

$$\eta_{risk} = 1 + \alpha_{risk} \cdot risk_i$$

α_{risk} is the weight that shows the level of Impact of consumers' inherent risk aversion on their exit decisions. To ensure the model produces results that match empirical observations and theoretical expectations, the exit rate of high-risk-aversion consumers should be higher than that of low-risk-aversion consumers. Therefore, after calibrating using the corresponding risk value α_{risk} , is set to 0.56.

c. Dynamic $\delta(m)$:

The definition of $\delta(m)$ is:

$$\delta(m) = \delta_0 + \alpha_{reinfect} \cdot \frac{I(m)}{N}$$

δ_0 is the basic return rate, representing naturally re-entering collaboration due to memory decay and favoring reciprocity

without external influence. It is set to be 0.01/month based on studies on natural retention rate in customer relationship management [14]. $\alpha_{reinflect} \cdot \frac{I(m)}{N}$ shows the attraction that other consumers' collaboration has on withdrawn consumers. $\alpha_{reinflect}$ quantifies such an effect; according to Ash's study on conformity behavior in social psychology, the value of $\alpha_{reinflect}$ is set to be 0.01.¹⁵

- A bilevel model between consumer collaboration and VPP profit:

Bilevel optimization models simulate decision-making in hierarchical settings and are widely used by public and private organizations in large-scale planning. The model consists of an upper level, the leader, which proposes solutions to optimize its goals, and a lower level, the follower, which responds to every proposed solution and reacts accordingly. The reaction of the lower layer will affect the final result of the upper layer's optimization.

a. Upper optimization level — VPP's profit maximization:

VPP acts as a price-maker, providing incentives to increase the consumer base and maximize profits.

$$\max \text{profit} = \sum_{m=1}^{12} \left[\sum_{i \in I(m)} p_i \cdot (\lambda_{\text{market}}(m) - \text{incentive}_i(m) - b_{op}(m)) \right]$$

$\lambda_{\text{market}}(m)$ is the market electricity price, $\text{incentive}_i(m)$ is the proposed incentives, $b_{op}(m)$ is the operation cost.

b. Lower level — SIR model:

The definition of the SIR model and its rates are the same as in the section Development of the dynamic SIR model. After the initial state of the cooperative group is set, the proposed incentive values will be input into the SIR model, which will report back the corresponding $I(m)$ and p_i values to the upper level. To quantify such an effect, whose value is set to be 0.01.

Solution to the proposed bilevel model:

The researchers use Python to call the HPSO-SA (Hybrid Particle Swarm Optimization - Simulated Annealing) algorithm, which can effectively solve the proposed complex bilevel problem.

PSO, particle swarm optimization (PSO), is a commonly used method to solve bilevel models that deal with a single decision maker and a follower.¹⁶ Both the leader and follower react optimally, where the follower has output y made based on the leader's solution x . PSO generates an initial population of random solutions and iteratively updates them to search for the optimal solution.¹⁶ The optimal solution is identified by its optimal fitness, which corresponds to the objective function value. Solutions are represented by particles flying in the space and move with the current optimal particles. Particles adjust their trajectory according to two references: the best position in their own history, p_{best} , and the current best position in the swarm, g_{best} . Each time PSO locates a better particle, the p_{best} and g_{best} will update accordingly. The updating rule for each particle is the following:¹⁶

$$\begin{aligned} \text{Updated velocity} : V_{\text{new}} &= wV_{\text{old}} + c_1r_1(p_{\text{best}} - X_{\text{old}}) + c_2r_2(g_{\text{best}} - X_{\text{old}}) \\ \text{Updated velocity} : X_{\text{new}} &= X_{\text{old}} + V_{\text{new}} \end{aligned}$$

Above, w represents the inertia weight, c_1 and c_2 are learning factors for cognitive and social influence, respectively, and r_1 and r_2 are two random numbers evenly distributed in $[0,1]$. After updates, PSO will run through another iteration and repeat the process until it reaches the maximum number of iterations and minimal improvement.

The proposed model has great complexity: first of all, it is a classical bilevel model where the upper level's incentive is closely connected to the lower level's cooperation rate; secondly, the heterogeneity within the consumer base causes effective incentives to be different for different consumers; thirdly, given the market uncertainty, the VPP should adjust incentive prices in real time. Therefore, the variables in the optimization problems have 72 dimensions:

$$\text{DecisionVariableDimensions} = 3(\text{categories}) \cdot 24(\text{months}) = 72\text{Dimensions}$$

In such a high-dimensional decision space, conventional PSO can hardly solve the problem effectively. Therefore, the researchers developed an improved PSO.

Basic PSO can easily get stuck in local optimals and affect the accuracy of the final outcome. Therefore, researchers integrate simulated annealing (SA) into PSO to occasionally accept worse particles, potential candidates for g_{best} , and compare them with the optimal g_{best} particle. If candidates have better fitness than the original g_{best} , the new g_{best} is calculated based on the superior candidate. If the candidates have worse fitness, accept it as the new g_{best} with probability:¹⁷

$$P = e^{-\frac{\Delta}{T}}$$

, where Δ is the fitness difference, and T is the current cooling temperature.

In addition, to illustrate the dynamic exploration of PSO, researchers process w , c_1 , and c_2 into three linear equations:

$$\begin{aligned} w &= w_{\text{max}} - \frac{(w_{\text{max}} - w_{\text{min}}) \cdot k}{K_{\text{max}}} \\ c_1 &= c_{1\text{max}} - \frac{((c_{1\text{max}} - c_{1\text{min}}) \cdot k)}{K_{\text{max}}} \\ c_2 &= c_{2\text{min}} + \frac{((c_{2\text{max}} - c_{2\text{min}}) \cdot k)}{K_{\text{max}}} \end{aligned}$$

, where k is the iteration number, and K_{max} is the maximum iteration number. In this way, at the first few iterations, larger w promotes particles to explore a broader region, and larger c_1 encourages particles to broadly search for a solution and avoid early convergence; as the iterations process, smaller w allows particles to explore more keenly and precisely in the optimal region, and larger c_2 enables the swarm to quickly narrow down to the best solutions areas for high precision.

Therefore, according to the design above, the HPSO-SA solution will run the lower-level SIR model, output the fitness of each particle (representing different incentives), update g_{best} and p_{best} , and each particle's position and velocity, and adjust the inertia weight and learning factors. Meanwhile, apply simulated annealing to the g_{best} particle to refine it, freeing the solution from local optima. Finally, when maximum iterations are reached and minimal improvements are made, output the optimal incentives and SIR curves, visualizing the calculation results.

In the VPP profit-maximization scenario, the profit-maximization function provides random incentives (particles) as input to the lower SIR model.

Below is the definition of fitness:

$$F_{\text{fitness}} = \text{Profit}_{\text{total}} - n_1 \cdot \max(0, CR_{\text{target}} - CR_{\text{final}})^3 + n_2 \cdot CR_{\text{avg}} - n_3 \cdot \sigma_{CR}$$

This equation uses the upper-level profit function as its core and integrates penalty and bonus terms to ensure that both profit and the cooperation ratio (I/N) are optimal and stable. Within the function, CR_{target} is the targeted cooperation ratio, CR_{final} is the total cooperation ratio at the end of the time range, σ_{CR} is the standard deviation that measures the stability of cooperation. n_1 , n_2 , and n_3 are all weighting coefficients.

Case Study:

a. Consumer base and heterogeneous categorization:

To demonstrate the proposed model's availability, researchers implemented it using market research data from 500 representative industrial enterprises, which were chosen randomly to illustrate the method's universality, aggregated by a virtual power plant in a city in southwest China. Researchers divided those 500 enterprises into 3 categories according to the categorization rule provided in the Model Derivation section, shown in the table below.

Table 2: Example enterprise categories. Based on the size and sensitivity, the sample consumers can be divided into 3 groups.

Categories	Electricity price sensitivity	Size	Customers population	Electricity consumption	Rationality k_r	Risk Reversion risk	Impact on society α_i
1	High	Medium	231	13.7	10.0	1.0	1.0
2	Medium	Small	129	12.7	8.0	1.3	1.3
3	Low	Large	140	14.2	6.0	0.8	0.7

b. PSO-SA Parameter Settings:

As mentioned earlier, this model is a relatively difficult bi-level optimization problem with 72 dimensions; therefore, the particle population is set to be larger than that of typical bi-level problems, which are usually 20-40. Other parameters for the PSO algorithm are based on He, Ma, and Zhang's studies on the influence of parameter settings on PSO performance, whereas the parameters of simulated annealing are based on the study by Park and Kim.^{18,19}

$N_{\text{pop}}=400$, $K_{\text{max}}=250$, $W_{\text{max}}=[0.4, 0.9]$, $c_1=c_2=2$, $T_{\text{initial}}=100$, attenuation coefficient $\alpha_{SA}=0.95$, bound = $[0.00035, 0.0045]$, $\text{velocity}_{\text{max}}=0.15$

c. Initial cooperation state settings:

$S = 400$, $I = 80$, $R = 20$

d. Net profit per month without incentives (Table 3):

The net profit is obtained through market research on the virtual power plant operator in western China.

$$\text{Net Profit} = \lambda_{\text{market}}(m) - b_{\text{op}}(m)$$

Table 3: Net profit each month. The monthly net profits reflect the southwest area's reliance on hydropower generation, where months 1, 2, 3, 4, 5, 13, 14, 15, 16, 17 are high-water season with great electricity profit, months 7, 8, 9, 10, 11, 19, 20, 21, 22, 23 are low-water season with low electricity profit, and month 6, 12, 18, 24 are transition period in between.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Net Profit (yuan/kWh)	0.0085	0.0092	0.0096	0.0088	0.009	0.0062	0.0012	0.001	0.0012	0.001	0.0008	0.0065
Month	13	14	15	16	17	18	19	20	21	22	23	24
Net profit (yuan/kWh)	0.0085	0.0092	0.0096	0.0088	0.009	0.0062	0.0012	0.001	0.0012	0.001	0.0008	0.0065

e. Comparative Experiment:

In order to demonstrate the superiority of the designed model, the researchers use three methods to create the best incentive strategy for 2 years.

Method 1: Conventional operation of the VPP, where the VPP does not provide incentives to consumers.

Method 2: Propose fixed incentives for each category throughout the 2-year range by employing optimization algorithms.

Method 3: Uses the proposed bilevel model above, which incorporates dynamic incentives for different categories over time.

Results and Discussion

Comparison Between Three Methods:

Based on Figure 2, the third method clearly shows superiority in both the cooperation rate (CR) and profit. First of all, the result shows the necessity of applying incentives as both the second and the third methods surpass the targeted CR, 80%, while the CR of the first method is only 65.5%. This shows that giving consumers incentives is an effective way to solve cooperation problems. Additionally, comparing method 2 and method 3, the researchers conclude that the dynamic incentive is better for the balance between cooperation rate and VPP's profit: provided that the cooperation rate does not decline, dynamic incentives yield the highest profits. Therefore, even though incentives can effectively stabilize the consumer base, a fixed, rigid strategy can devour profits to maintain a high cooperation rate, whereas a flexible strategy ensures both by spatiotemporally refining the allocation of incentive budgets.

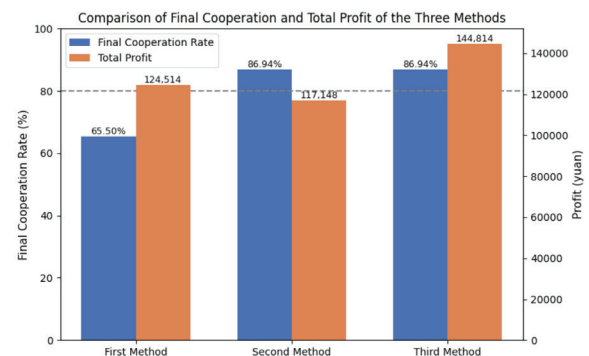


Figure 2: Comparison of final cooperations and total profits of the three methods. The third method reaches the highest profit and best balance between profit and cooperation.

Detailed Analysis of the Third Method:

After the PSO-SA calculation of the model designed through Python, the target VPP exhibits significant financial profit and synergy. First of all, based on Figure 3, VPP exhibits a great and stable participation rate with an average of 87%, which is successfully higher than the target rate 80%. This shows the positive effect of such an incentive strategy on the VPP's consumer base. In addition, consumers in different categories exhibit different participation rates, as shown in Figure 4. The larger size consumer (High) has significantly higher participation, while the Small size consumer (Low) and the Medium size consumers (Med) have similar participation.

The differences in participation are due to differences in the electricity consumption scale and the sensitivity to incentives: Large consumers have a larger scale and lower sensitivity, and therefore exhibit a higher and more stable participation rate; even though small consumers have a smaller sensitivity than the medium-sized consumers, they have a lot less energy consumption and population, which explains why they have a slightly lower participation rate and a higher amount of average incentive in Figure 5. This shows the necessity of including consumer heterogeneity in the model, which guides the VPP in offering different and effective incentives shown in Figure 7. The result also shows how the model is designed to create adaptive and granular budgeting strategies for VPPs. Based on Figure 3, the average total incentives are highest at months 14 and 15, which have the highest next profit. The model, therefore, shows how VPP should appropriately increase the amount of incentives when the market price increases to balance its profit and the participation rate. The strategy creates economic profit for both the VPP and consumers: under such an incentive system, the consumers are able to save 0.0037 ¥/MW, and the total profit generated by the VPP is ¥144,814.23 based on Figure 6.

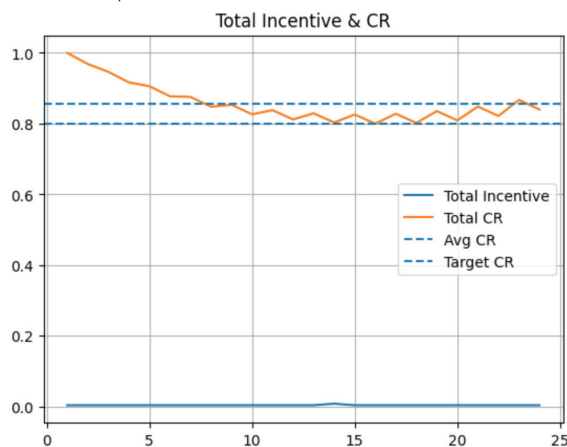


Figure 3: Total cooperation rates (CR) and total incentives per month. Total cooperation rate remains above target throughout the studied time range, with a higher cooperation rate and incentive during the high-water season.

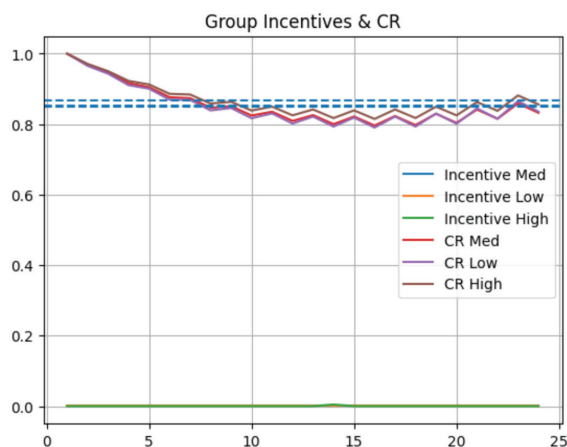


Figure 4: Categorical participation rates (CR) and categorical incentive amount (MW/yuan). The cooperation rate of big-sized consumers exhibits a significantly larger cooperation rate than that of smaller consumers.

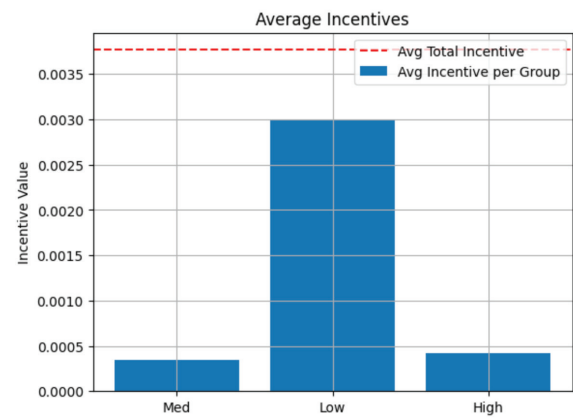


Figure 5: Enlarged version of total and categorical incentive amount (MW/yuan). Small-sized consumers require a higher incentive compared to bigger consumers.

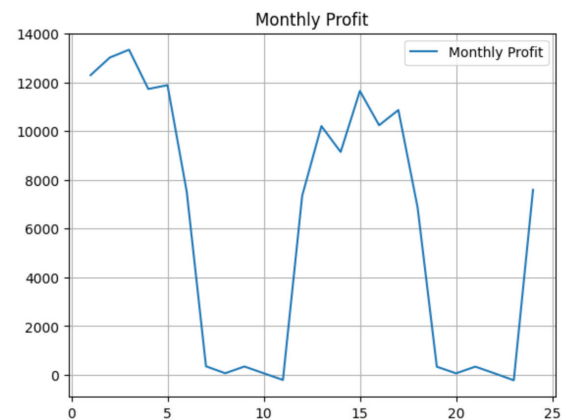


Figure 6: Monthly profit of the virtual power plant. Monthly total profit corresponds to the profit differences between high-water season and low-water season and sums up to a positive annual profit.

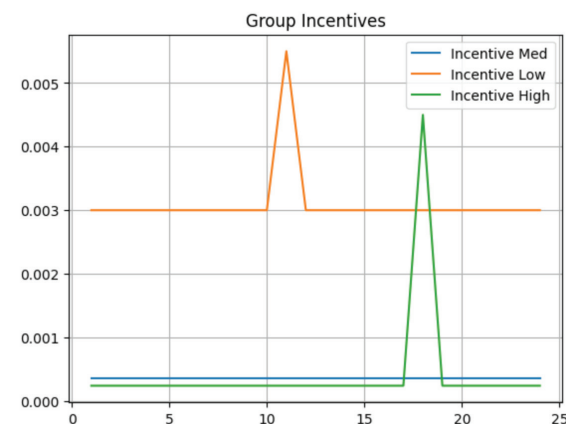


Figure 7: Monthly incentives for each category. The incentive strategy unevenly allocates incentives among heterogeneous consumer groups to meet the requirements of diverse consumers in a personalized manner.

■ Conclusion

Based on the result above, the proposed bilevel system successfully developed an incentive strategy for VPP's long-term profitability and stability of cooperative relationships. The strategy also emphasizes heterogeneity within the consumer base and responds with different incentive amounts, a factor largely overlooked in previous studies. The modified PSO-SA

algorithm serves to accurately solve such complicated models and provide robust technical support. Therefore, the model provides not only a deep and novel understanding of coordinating distributed resources, but also an actionable and insightful practical guide for VPP's operation under modern electricity systems. However, research on VPP market pricing extends far beyond this point. Future research can effectively increase the models' accuracy by specifically improving the categorization system in order to incorporate more diverse consumers and track subtle interactions between them. The multipliers used in categorification also vary depending on the specific virtual power plant consumer groups: the proposed model is proven to be viable, yet real applications need calibration. Additionally, specific methods to implement different incentives remain vague: future studies are needed in order to avoid customer disputes resulting from price discrimination between different categories.

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■ Authors

Ke Liu, a junior at Valor Christian High School, is enthusiastic about electrical engineering and applied calculus. She wants to study engineering and mathematics in college and further research on renewable energy and its implementation.