

Motorsport Brake Systems: Numerical Simulation and Analysis of Heat Distribution in Perforated Brake Discs

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ABSTRACT: In motorsport, the cooling of the braking system is very important. Sustained high temperatures, resulting from repeated braking under high loads or at high speeds, cause a highly undesirable temporary and sudden reduction in braking power. To reduce these effects, brake discs and pads in racecars are typically made up of Carbon-carbon composite Materials; however, they too can experience brake fade, resulting in decreased braking performance, with sustained high temperatures leading to structural damage to the brake disc, warped brake discs due to uneven thermal expansion, and the boiling of the brake fluid causing the brake to feel spongy and unresponsive. Additionally, at these high temperatures, the brake disc surface may oxidize, causing the dust from the friction of the brake disc as well as surrounding components to possibly ignite at temperatures beyond 1200 °C, causing a fire, thus preventing the racecar from continuing to race. This research investigates the cooling of perforated and non-perforated brake discs by numerically simulating the heat equation in one, two, and three dimensions, using convective boundary conditions at the edges of the simulated object (including at its boundaries). Results indicate that geometry, specifically the surface area of the brake disc due to the number of perforations, strongly influences cooling rates.

KEYWORDS: Mechanical Engineering, Heat Equation Modeling, Finite Difference Method, Convective (Robin) Boundary Conditions, Brake Disc Thermal Simulation.

■ Introduction

In motorsport, braking systems are subjected to extreme thermal loading, with rapid, repeated high-speed deceleration events generating substantial frictional heat at the brake pad-brake disc interface.¹ This heat generation occurs consistently throughout long race durations, leading to continuous cycles of heating and cooling. Therefore, effective heat dissipation is essential for maintaining braking performance and preventing brake fade, a phenomenon in which braking torque is reduced at elevated temperatures due to temperature-dependent changes in the frictional behavior of the brake disc-brake pad interface (and the brake disc surface, which may become glazed or smooth due to friction), which reduces braking performance.² As such, thermal behavior is a critical engineering consideration in motorsport, motivating detailed analysis of how braking systems dissipate heat under sustained loading.

In this research, carbon-carbon composite brake materials were used in the modeling due to their persistent application in most international racing disciplines, such as Formula 1, WEC (including LMH, LMDh and LMGT3), and Indycar,^{3,4} and their material properties resulting from its structure (carbon fibers embedded in a carbon matrix, hence the name):⁵ high thermal diffusivity, low density, and superior resistance to thermal degradation (compared to traditional metallic-based brake systems and carbon ceramic brake systems found in high-end sports cars). These properties allow carbon-carbon composite components to withstand repeated temperature cycling (repeated cooling to low temperatures and heating to extremely high temperatures) more effectively than metallic-based systems.⁶ As such, highlighting why carbon-carbon composite

braking material properties were chosen to be used in these modeled simulations.

Mathematical modeling provides a meticulous foundation for understanding how heat diffuses (propagates) through brake pads and discs under real-world racing conditions. The governing model used throughout this study is the heat equation, because it directly represents the fundamental physical mechanism of thermal diffusion: differences in temperature within a solid drive conduction from hotter regions to cooler regions.⁷ This is the same behavior expected in brake pads and discs after frictional heating during braking introduces thermal energy at the surface.⁸ Once heat enters the brake discs (and brake pads), the internal redistribution of temperature is dictated by conduction. The heat equation provides a direct and established way to represent that redistribution over time and space, also allowing the spatial intensity of heat (hotspots and gradients) to be predicted over time. For these simulations, the heat equation also provides a malleable foundation for testing how geometrical conditions influence temperature profiles, essential when modeling perforated brake discs in motorsport braking systems.

This paper explicitly considers the heat equation in one, two, and three dimensions.⁹

The one-dimensional form serves as a baseline conduction case, useful for validating numerical behavior when heat flow is approximated along a single direction in space:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ is the temperature along the rod, (dependent on position x and time t) and α is the thermal diffusivity of the material (Carbon-carbon composite). The modeling is then

extended to two dimensions to capture temperature variation across a planar domain:

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

where $u(x,y,t)$ is the temperature along the rod and α is the thermal diffusivity of the material (Carbon-carbon composite).

The three-dimensional form is then included to account for conduction in the depth direction:

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

where $u(x,y,z,t)$ is the temperature along the rod, and α is the thermal diffusivity of the material (Carbon-carbon composite).

NOTE: α is calculated by the equation $\alpha = \frac{k}{\rho C_p}$ where k is the thermal conductivity, ρ is the density, and C_p is the specific heat capacity.

Because brake discs are circular in geometry (flat cylindrical shape), the study further established the transformation from Cartesian coordinates (x,y,z) to cylindrical coordinates (r,θ,z) using the coordinate relations $x=r\cos\theta$, $y=r\sin\theta$, and $z=z$. By applying the multivariable chain rule, the Laplacian operator becomes:

$$\nabla^2 u = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$$

Boundary conditions play a critical role in predicting brake-disc temperatures, as the heat equation only describes how temperature diffuses within the brake disc itself, not specifying how the brake disc interacts with its surroundings. In motorsport applications, heat is continually removed at the boundaries: exposed faces (and edges) exchange heat with the surrounding fluid, and hollow structures within the disc (such as perforations in the disc) act as additional cooling boundaries. Thus, in order to produce a realistic simulation of the brake disc temperature distribution, it is imperative that the boundary conditions are assigned.¹⁰ Three boundary conditions were analyzed. Dirichlet (fixed temperature) boundary conditions assume the boundary is held at a constant temperature. This provides a theoretical lower bound on temperature but overestimates cooling compared with real brake discs, whose surfaces heat up significantly under load. Neumann boundary conditions assume zero heat flux across the boundary; thus, heat cannot leave the system. While this may be useful for testing pure conduction behavior or establishing an upper bound for temperature retention, it is not representative of braking systems in airflow because the parts of the braking system (notably the brake discs and brake pads) always lose heat to their surroundings. By contrast, convective (Robin) boundary conditions model heat loss in a way that closely matches real braking behavior: Brake discs cool primarily through convection to air, especially in motorsport, where airflow is manipulated and forced, and ventilation features are designed to accelerate the rate of heat removal. Convective conditions accurately reproduce this by linking heat flux to the temperature difference between the surface and the ambient environment, allowing the model to represent thermally active boundaries, including surface and internal circular openings, that remove heat

at a rate dependent on how hot the material is. Because this aligns with how brake components exchange heat with airflow and cooling channels, convective boundary conditions are the most physically realistic of the three. The convective (Robin) boundary condition is written as:¹¹

$$-k \frac{\partial u}{\partial n} = h(u - T_\infty)$$

where $\frac{\partial u}{\partial n}$ represents the outward normal temperature gradient at a surface. This normal derivative is physically meaningful both at the outer edges of the disc/plate and at the interior boundaries of the circular perforations for cooling.

To calculate solutions for realistic brake geometries, the heat equation was solved numerically using the finite difference method; spatial and temporal derivatives were approximated using an FTCS (Forward-Time Central-Space) discretization, and the resulting discrete system was advanced in time to obtain the temperature field.

Using this modeling framework, the brake disc is modeled as a 0.5 m x 0.5 m two-dimensional square plate in four configurations: no perforations, one 10 cm diameter circular perforation, 16 evenly spaced 1 cm perforations, or 25 evenly spaced 1 cm perforations. Convective boundary conditions are imposed at the perforation (hole) boundaries, promoting internal cooling, and at the outer edges, ambient cooling, and temperature fields are sampled at regular intervals over 1000 s. This will be expanded on in the methodology section.

The goal of this research is to analyze brake-disc thermal behavior through mathematical modeling of heat distribution under geometric constraints, focusing on how surface-area changes affect heat dissipation pathways. Notably, the perforations are not treated as a cosmetic feature but rather as a quantifiable geometric intervention: increasing the internal boundary length (and therefore effective cooling area) changes the temperature field by introducing additional locations where heat can be transferred to the surrounding fluid. To isolate this effect, a carbon-carbon composite brake disc with dimensions similar to those of a 2014 Mercedes W05 F1 car is modeled under 2 controlled configurations:

- 1) 1000 perforations of diameter 3 mm
- 2) 800 perforations of diameter 3 mm

With convective (Robin) boundary conditions applied to the entire surface exposed to the surrounding fluid. Outputs are generated at regular time intervals.

The 2014 Mercedes W05 F1 Car was chosen as it represents the turning point of Formula 1, bringing the historical motorsport into the modern era with its advanced hybrid power unit, molded by the regulations of that year.

This aim is motivated by the context that motorsport brakes undergo extreme, rapidly changing thermal loads, where large thermal gradients and high temperatures can induce brake fade (loss of braking power due to the excessively high temperature of the brakes) and where accurate heat-transfer understanding becomes a priority. Existing work discussed in this project includes analytical approaches (i.e., moving heat source simulations), experimental approaches (i.e., thermographic imaging), and FEM simulations in platforms such as ANSYS.^{1,12} However, the literature considered in this research

often does not closely examine brake-disc thermal interactions using simplified numerical solvers in a way that makes geometry-boundary-temperature relationships easy to interpret

Consequently, this research differs from several prior approaches in two main ways. First, it deliberately prioritizes a finite-difference framework, discretizing the (one, two, and) three-dimensional heat equation using an explicit FTCS formulation such that the link between the governing partial differential equation and the resulting temperature fields remains explicit. In the present work, interpretation is reported primarily on the central cross-section of the brake-disc domain to make spatial trends directly comparable across time. Second, rather than treating convection only on the external boundary, the study explicitly applies convective (Robin) cooling at internal perforation boundaries, as at the external boundaries, such that surface-area changes are evaluated through a physically meaningful mechanism, heat exchange with surrounding fluid, at every cooling interface.

Ultimately, the contribution of this study is a controlled numerical investigation, grounded in the heat equation and FTCS discretization, into how brake-disc relevant geometry reshapes volumetric heat distribution and cooling trends. By comparing different configurations of the brake disc under consistent boundary modeling, this work provides an interpretable basis for connecting geometric design choices to heat dissipation outcomes in carbon-carbon composite brake-disc analogs.

■ Methods

The finite difference method (FDM) was chosen in preference to the Euler method because the FDM is a complete and consistent numerical discretization of the heat equation,⁹ whereas the Euler method is only a time-integration rule.¹³ The governing heat equation pairs the temporal rate of change of temperature to the second-order spatial derivatives (the Laplacian) and thus requires a computable approximation not only for $\frac{\partial u}{\partial t}$ but also for $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 u}{\partial z^2}$. While forward Euler time stepping provides an approximation of the temporal derivative, it does not, by itself, deliver a mechanism to evaluate the spatial curvature terms that drive diffusion. By contrast, FDM replaces the spatial derivatives with grid-based finite-difference stencils, most commonly central differences, thereby converting the partial differential equation into an explicit algebraic update rule at each node. This discretization naturally leads to the FTCS scheme, which pairs forward-time stepping with central space differencing, resulting in a symmetric diffusion model consistent with isotropic (same temperature) heat spreading on a uniform mesh. However, it is important to note that the FTCS scheme employs a Taylor series expansion where higher-order terms are neglected. These neglected terms are the error of the large simulation that has compounded over long time ranges (over a large number of iterations), and thus this is an inherent uncertainty in this method.

1D baseline rod simulation:

Governing equation (1D Heat conduction with variable conductivity).

The one-dimensional temperature field $T(x,t)$, on a rod of length, L is governed by the transient heat conduction equation with spatially varying thermal conductivity $k(x)$:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k(x) \frac{\partial T}{\partial x} \right) + q(x)$$

where ρ is density (kgm^{-3}), c_p is specific heat capacity ($\text{Jkg}^{-1}\text{K}^{-1}$), $k(x)$ is thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$), and $q(x)$ is volumetric heat generation (Wm^{-3}). Because this paper only simulates the cooling of the brake discs, $q(x)=0$ such that the temperature decreases monotonically from the initial conduction under convective heat loss.

Geometry, grid, and material parameters:

A rod of length $L=1.0\text{ m}$ was discretized using $N_x=50$ grid points:

$$\Delta x = \frac{L}{N_x-1}, x_i = i\Delta x, i=0, 1, \dots, N_x-1$$

Carbon-carbon composite representative constants used in the numerical model were:

$$\begin{aligned}\rho &= 1780 \text{ kgm}^{-3} \\ c_p &= 2218 \text{ J kg}^{-1}\text{K}^{-1}\end{aligned}$$

Thermal conductivity was modeled as linearly varying along the rod

$$k(x) = k_0(1 + 0.5\frac{x}{L}), k_0 = 86.3 \text{ Wm}^{-1}\text{K}^{-1}$$

Such that conductivity increases from 86.3 to 129.45 $\text{Wm}^{-1}\text{K}^{-1}$ across the domain.

Boundary conditions (convective boundary conditions):

Convective heat exchange with ambient air was imposed at both ends of the 1D rod. With ambient temperature T_∞ and heat transfer coefficient h , the end conditions are:

$$\text{At } x=0, k(0)\frac{\partial T}{\partial x}(0,t) = h(T(0,t) - T_\infty)$$

$$\text{At } x=L, -k(L)\frac{\partial T}{\partial x}(L,t) = h(T(L,t) - T_\infty)$$

The simulations used $T_\infty=23\text{ }^\circ\text{C}$, and $h=60\text{ Wm}^{-2}\text{K}^{-1}$.

To represent a purely cooling scenario, the rod was initialized uniformly above ambient:

$$T(x,0) = T_0$$

with T_0 set at 1000 $^\circ\text{C}$ (to allow for an analyzable difference between, which is higher than T_∞). With $q(x)=0$, this ensures the solution cools towards ambient over time rather than heating.

Numerical method: flux-conservative finite differences with FTCS time stepping:

The partial differential equation was solved numerically using the finite difference method on a uniform grid. A flux-conservative discretization was used for the variable-conductivity diffusion term by defining midpoint conductivities:

$$k_{i+0.5} = \frac{k_i + k_{i+1}}{2}$$

For interior nodes, $i=1, \dots, N_x-2$, the spatial operator was approximated as

Time integration was performed using an explicit Forward-Time Central-Space (FTCS) update:

$$T_i^{n+1} = T_i^n + \frac{\Delta t}{\rho c_p} \left[\frac{1}{\Delta x^2} (k_{i+0.5}(T_{i+1}^n - T_i^n) - k_{i-0.5}(T_i^n - T_{i-1}^n)) + q_i \right]$$

For the cooling-only case, $q_i = 0$.

Implementation of convective boundary conditions:

To enforce the convective boundary conditions within the explicit scheme, a ghost-node method was used. At $x=0$, the derivative was approximated by a centered difference:¹⁴

$$\frac{\partial T}{\partial x}(0, t) \approx \frac{T_1 - T_{-1}}{2\Delta x}$$

Substituting into $k(0)T_x(0, t) = h(T_0 - T_\infty)$ gives the left ghost value T_{-1} in terms of interior values:

$$T_{-1} = T_1 - \frac{2\Delta x h}{k(0)}(T_0 - T_\infty)$$

Similarly, at $x=L$,

$$\frac{\partial T}{\partial x}(L, t) \approx \frac{T_N - T_{N-2}}{2\Delta x}$$

And substitution into $k(L)T_x(L, t) = h(T_{N-1} - T_\infty)$, yields the right ghost value T_N . These ghost values were then used in the same flux-conservative formula to update the boundary nodes $i=0$ and $i=N_x-1$, ensuring the ends remain thermally active (the temperatures are not fixed but vary according to convection strength h).

Time stepping, stability control, and reported output times:

A uniform timestep Δt was used with explicit FTCS. Because explicit diffusion schemes are conditionally stable, timestep selection was controlled relative to grid spacing and the maximum effective diffusivity

$$\alpha_{\max} = \frac{\max(k)}{\rho c_p}$$

and Δt was kept sufficiently small to satisfy the standard FTCS stability requirement, as $\Delta t \leq \Delta t_{\text{stable}}$.¹⁵

Five temperature profiles were extracted and plotted at 30-second intervals such that the changes in temperature distribution would be more apparent and easier to identify.

All computations were implemented in Python (and ran in Google Colab), using array-based numerics and plotted as two-dimensional line graphs of $T(x, t)$ versus x . The rod domain was plotted over $0 \leq x \leq 1$ m for ease of simulation.

2D Carbon-carbon composite Plate cooling with convective (Robin) boundaries:

A two-dimensional transient heat-diffusion model was used to study cooling in a 0.5×0.5 m square-shaped carbon-carbon composite plate with convective heat loss at the outer perimeter and at the walls of the circular perforations within the boundaries of the plate. The plate temperature field $T(x, y, t)$ was governed by the 2D heat equation:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

where α is the thermal diffusivity.

Geometry and perforation configurations:

A square plate of side length 0.5 m was used for all 2D cases. Four configurations were simulated:

- 1) Unperforated plate, with a continuous solid domain with convection only at the edges of the plate
- 2) Singular central perforation of diameter 10 cm on the plate, with convection at the edges and perforation boundary

3) Perforated plate with 16 evenly distributed perforations of diameter 1 cm on the plate, with convection at the edges and the perforation boundary

4) Perforated plate with 25 evenly distributed perforations of diameter 1 cm on the plate, with convection at the edges and the perforation boundary

Boundary conditions:

Convective heat exchange was imposed using a convective (Robin) boundary condition:

$$-k \frac{\partial T}{\partial n} = h(T - T_\infty)$$

Applied on all outer edges of the plate and on internal boundaries along the perforation boundaries, where h is the convection coefficient, and n is the outward normal.

In the discrete scheme, convective boundary conditions were implemented using a Ghost-node method (where a virtual node outside the domain is used). To apply convective boundary conditions on a Cartesian grid without explicitly meshing curved boundaries, the boundary flux condition is enforced by replacing a missing neighbor (outside the domain) with an equivalent 'ghost' temperature. This is implemented in the form:¹⁶

$$T_{\text{ghost}} = T_{\text{cell}} - \beta(T_{\text{cell}} - T_\infty), \quad \beta = \frac{h\Delta x}{k}$$

This is the discrete ghost-node expression consistent with enforcing the convective boundary condition at the boundary.

This is applied at the outer edges and the perforation boundaries in the simulations.

Numerical discretization: FTCS finite difference:

The 2D plate was discretized on a uniform Cartesian grid over $[0, L_x] \times [0, L_y]$ with $\Delta x = \Delta y$. The temperature was advanced using an explicit FTCS scheme:¹⁷

$$T_{ij}^{n+1} = T_{ij}^n + \alpha \Delta t \left(\frac{T_{i+1,j}^n - 2T_{ij}^n + T_{i-1,j}^n}{\Delta x^2} + \frac{T_{i,j+1}^n - 2T_{ij}^n + T_{i,j-1}^n}{\Delta y^2} \right)$$

with neighbor terms replaced by the convective ghost values whenever a boundary is encountered.

Stability condition:

The explicit diffusion scheme requires a time step satisfying the standard 2D criterion ($r_x + r_y \leq 0.5$); for $\Delta x = \Delta y$, this reduces to $r \leq 0.25$ where $r = \frac{\alpha \Delta t}{\Delta x^2}$.¹⁸

This was enforced in the simulations using:

$$\Delta t = 0.8 \Delta t_{\max}, \quad \Delta t_{\max} = 0.25 \frac{\Delta x^2}{\alpha}$$

Simulations:

Cells inside the perforations were treated as void and held at ambient (296.15 K):

$$T(x, y, t) = T_\infty \text{ for perforations.}$$

This provides a strong cooling sink and, combined with Robin handling at the solid-void interface, enforces internal convective heat removal at perforation walls.

High-resolution perforated cases (where circular perforations were modeled) were used $N_x = N_y = 81$ for the 16-perforation case for improved circularity of perforations on a Cartesian mesh.

Sampling of the thermal distribution of the four plates occurred at regular 30-second intervals, using a color scale for ease of analysis. 30 seconds was chosen, as it provided sufficient time for the temperature to change sufficiently such that a difference could be seen, but not too much time such that the temperature change would be too extreme.

To quantify the cooling rate, the solid-only mean temperature was computed at each time step:^{19,20}

$$\bar{T}(t) = \frac{1}{A_{solid}} \int_{\Omega_{solid}} T dA \approx \frac{1}{N_{solid}} \sum_{(i,j) \in solid} T_{i,j}$$

This was then plotted onto a graph for all cases (refer to Figures 2, 3, 4, and 5), allowing direct comparison.

All simulations and post-processing were implemented in Python using NumPy for array operations and Matplotlib for plotting/exporting figures (including multi-page PDF outputs). The scripts were executed in Google Colab to ensure reproducibility and consistent runtime settings.

Modeling assumptions

In this model, the material properties a and k are treated as constant, and heat transfer to the environment is represented solely by convective boundary conditions with constant h and T_{∞} , with radiation and contact resistance neglected. The fluid within the perforations is treated as an ambient reservoir at T_{∞} , while convective exchange at each perforation boundary is enforced on the solid side via the ghost-node method. Material properties within the plate are assumed homogeneous and isotropic, and no internal heat generation is included.

Parameters:

In this section, I list the parameters used for the 2D simulation. These values all reflect the properties of the carbon-carbon composite material that motorsport brake discs are composed of.

- 1) 2D Plate dimensions 0.5×0.5 m
- 2) Carbon-carbon composite density,²¹ $\rho = 1780.0 \text{ kgm}^{-3}$
- 3) Specific heat capacity,²² $C_p = 2218 \text{ Jkg}^{-1}\text{K}^{-1}$
- 4) Thermal conductivity,²³ $k = 86.3 \text{ Wm}^{-1}\text{K}^{-1}$
- 5) Convective heat transfer coefficient, $h = 100 \text{ Wm}^{-2}\text{K}^{-1}$
- 6) Initial plate temperature, $T_{init} = 1273.15 \text{ K}$
- 7) Surrounding ambient temperature, $T_{\infty} = 296.15 \text{ K}$

3D Transient brake disc cooling model:

The brake disc was modeled as a 3-dimensional annular solid using cylindrical coordinates, using the standard coordinate relations $x=r\cos\theta$, $y=r\sin\theta$, $z=z$, and discretized on a structured (r, θ, z) to support an explicit FTCS finite-difference time integration. Heat transfer within the disc was governed by the transient heat equation

$$\frac{\partial T}{\partial t} = \alpha(T) \nabla^2 T, \text{ where } \alpha(T) = \frac{k(T)}{\rho c_p(T)}$$

The specific heat capacity, $C_p(T)$ was implemented as a piecewise-linear interpolation table, using the three tabulated points at 473.15 K, 1273.15 K, 1773.15 K. The thermal conductivity, $k(T)$ was also treated as temperature-dependent using a piecewise-linear table, and the thermal diffusivity was updated consistently as $\alpha(T) = \frac{k(T)}{\rho c_p(T)}$.

Geometry of the brake disc:

The disc geometry was defined by the outer diameter $D_o = 0.278$ m, inner diameter $D_i = 0.1816$ m, and thickness $H = 0.028$ m, giving $R_o = D_o/2 = 0.139$ m and $R_i = D_i/2 = 0.0908$ m. Because the inner radius of the brake disc is not specified in the 2014 FIA Regulations,²⁴ and F1 teams prefer to keep their designs secret to maintain a competitive edge, D_i , and thus R_i , was estimated from an image of the Mercedes W05 at the German Grand Prix.²⁵ Through-radial perforations were represented with an estimated perforation diameter $d_p = 0.003$ m and perforation depth set equal to the full radial width of the annulus, $L_{depth} = R_o - R_i$.²⁶

Material model:

The disc is treated as an effective isotropic carbon-carbon composite solid with constant density $\rho = 1780 \text{ kgm}^{-3}$ and temperature-dependent heat capacity and conductivity. The specific heat capacity, $C_p(T)$ was implemented as a piecewise-linear interpolation table, using the three tabulated points at 473.15 K, 1273.15 K, 1773.15 K.

The material properties were updated one per global time-step using the volume-weighted mean temperature $T_{mean}(t^n)$ to maintain internal consistency in the transient property model $c_{p,eff}^n = c_p(T_{mean}^n)$, $k_{eff}^n = k(T_{mean}^n)$, $\alpha_{eff}^n = \frac{k_{eff}^n}{\rho c_{p,eff}^n}$.

Initial conditions:

The disc is initialized at a uniform temperature representing the post-braking thermal state of the brake disc of a 2014 Mercedes W05 F1 car exiting turn 2 at the Autodromo Nazionale Monza Circuit in Italy, where $T(r, \theta, z, \theta) = T_0 = 1000^\circ\text{C} = 1273.15 \text{ K}$. Ambient air temperature is prescribed as $T_{\infty} = 23^\circ\text{C} = 296.15 \text{ K}$.

This uniform initial temperature assumption provides a controlled starting condition, allowing the subsequent cooling to be attributed primarily to conduction, convection, and perforation-enhanced heat loss during the 15 second acceleration window.

Boundary conditions:

Cooling to airflow is represented using convective boundary conditions, which model heat loss proportional to the temperature difference between the solid and the surrounding air at convective boundaries. Accordingly, convective boundary conditions were applied on the inner bore, $r=R_i$; Outer rim, $r=R_o$ and both disc faces, $z=0$ and $z=H$, in the standard form:

For numerical implementation within the explicit FTCS scheme,²⁹ the convective boundary condition

$$-k_{eff} \frac{\partial T}{\partial n} = h(t)(T - T_{\infty})$$

is enforced using a ghost-cell formulation, where n is the outward normal, T_{∞} is ambient temperature, and $k_{eff} = k(T_{mean})$ is the effective conductivity evaluated at the volume-weighted mean temperature at each time step. This ghost-cell approach converts the boundary heat-flux relation into an equivalent virtual temperature just beyond the computational domain, enabling the same centered-difference Laplacian stencil to be applied at boundary nodes whilst still imposing convective heat loss.

Radial boundaries ($r=R_i, r=R_o$):

At the inner bore, $r=R_i$, the outward normal points in the $-\hat{e}_r$ direction, so $\frac{\partial T}{\partial n} = -\frac{\partial T}{\partial r}$. The convective boundary condition becomes

$$-k_{eff}\frac{\partial T}{\partial n} = h_{in}(t)(T - T_{\infty})$$

Using a centered difference at the boundary node with ghost temperature T_{-1} gives

$$\frac{T_1 - T_{-1}}{2\Delta r} = \frac{h_{in}(t)}{k_{eff}}(T_0 - T_{\infty}) \rightarrow T_{-1} = T_1 - 2\Delta r \frac{h_{in}(t)}{k_{eff}}(T_0 - T_{\infty})$$

At the outer rim $r=R_o$, the outward normal is $+\hat{e}_r$, thus $\frac{\partial T}{\partial n} = \frac{\partial T}{\partial r}$ and

$$-k_{eff}\frac{\partial T}{\partial n} = h_{out}(t)(T - T_{\infty})$$

With boundary node T_{Nr-1} and ghost node T_{Nr}

$$\frac{T_{Nr} - T_{Nr-2}}{2\Delta r} = -\frac{h_{out}(t)}{k_{eff}}(T_{Nr-1} - T_{\infty}) \rightarrow T_{Nr} = T_{Nr-2} - 2\Delta r \frac{h_{out}(t)}{k_{eff}}(T_{Nr-1} - T_{\infty}) \quad (30)$$

These ghost values are substituted into the FTCS Laplacian evaluation such that the radial diffusion term remains second-order in space whilst consistently applying the convective flux at both radial edges.

Axial (face) boundaries:

Similarly, convection on the disc faces is enforced using ghost temperatures with outward normal $-\hat{e}_z$ at $z=0$ and $\hat{e}_z$ at $z=H$. The ghost values used in the centered difference are

$$T_{z,-1} = T_{z,1} - 2\Delta z \frac{h(t)}{k_{eff}}(T_{z,0} - T_{\infty}), \quad T_{z,N_z} = T_{z,N_z-2} - 2\Delta z \frac{h(t)}{k_{eff}}(T_{z,N_z-1} - T_{\infty}) \quad (31)$$

Consistency with the time-dependent convection model:

At each time step, $h(t)$, $h_{in}(t)$ and $h_{out}(t)$ are evaluated from the prescribed velocity-based convection model, and k_{eff} is obtained from the temperature-dependent material model. This ensures that the applied boundary heat flux remains consistent with both the time-varying airflow during the acceleration of the F1 car and the temperature dependence of the carbon-carbon composite properties, specifically thermal conductivity, specific heat capacity, and thermal diffusivity.

Ghost-cell temperature is recomputed each time step using the current $h(t)$ and k_{eff} and substituted into the boundary Laplacian stencil.

Perforation-enhanced cooling modeling:

Because the brake discs of 2014 F1 cars contain perforations (Ventilation holes), additional cooling is included through 2 physically motivated mechanisms without explicitly meshing each perforation.

1) The internal perforation-wall convection as a distributed sink term

The increased internal surface area provided by the perforation walls was modeled as a volume-averaged (represented as an effective volumetric cooling term) convective heat-loss term applied throughout the solid.²⁸

$$\frac{\partial T}{\partial t} = -\beta(t)(T - T_{\infty}), \quad \beta(t) = \frac{h(t) A_{perforations}}{\rho c_p (T_{mean}) V_{solid}}$$

where $\beta(t)$ is the time-dependent volumetric cooling rate coefficient, $A_{perforations}$ is the total lateral area of all cooling perforations (where the endcaps are neglected).

($A_{perforations} = N_{perforations} \pi d_h L_{depth}$) and V_{solid} is the net solid material volume of the disc ($V_{solid} = V_{annulus} - V_{perforations}$).

$$V_{annulus} = \pi(R_o^2 - R_i^2)H, \quad V_{perforations} = N_{perforations} \pi \left(\frac{d_h}{2}\right)^2 L_{depth}$$

This sink term (added volumetric cooling term) is integrated analytically over each Strang half step $\Delta t/2$

$$T \leftarrow T_{\infty} + (T - T_{\infty})e^{-\beta(t^n)\frac{\Delta t}{2}}$$

2) Perforation-induced enhancement of convection at the radial-edge:

For perforations (through the radius), the perforations also increase airflow interaction near both radial edges.¹ This is represented by boosting the effective convection coefficient on the inner bore and outer rim by distributing the perforation (hole) area evenly between the edges:

$$h_{edge}(t) = h(t) \frac{A_{edge} + 0.5A_{perforations}}{A_{edge}}, \quad A_{edge}(R) = 2\pi RH$$

Applied separately at $R=R_i$ and $R=R_o$. The face convection coefficient $h(t)$ used on $z=0$ and $z=H$ was not edge boosted. A modeling approximation, the factor $0.5A_{perforations}$ assumes that the through-radial perforations contribute equally to enhanced convection at the inner and outer radial edges. (The face coefficient $h(t)$ applied on $z=0$ and $z=H$ was not edge boosted, matching the implementation).

Together, the convective boundary conditions and the perforations allow the model to represent both external forced convection and perforation-enhanced heat removal.

Airflow convection-coefficient model:

To represent increasing forced convection during acceleration out of turn 2 in Monza, the convection coefficient is modeled as a function of time via a velocity series, prescribing a piecewise-linear velocity profile, defined at 1 second intervals over 0-15 s. The prescribed velocity (V/kmh^{-1}) values over 0-15 s is shown in the table below.³²

Table 1: Mercedes W05 2014 F1 car Speed-Time relationship out of turn 2 Monza.

s	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
v	67	80	138	192	225	250	268	283	290	300	310	315	325	328	332	332

These values were used as a representative for the magnitude of forced convection.

A power law correlation maps the changing velocity to convection:³³

$$h(t) = h_{ref} \left(\frac{v(t)}{v_{ref}} \right)^{0.8}$$

with $h_{ref} = 100 Wm^{-2}K^{-1}$ at $V_{ref} = v(0)$. This provides a simple monotonic increase in convection strength during acceleration, consistent with the modeling approach of forced-air cooling.

Numerical method:

The computational domain for the brake disc was defined in cylindrical coordinates (r, θ, z) over the annular region $R_i \leq r \leq R_o$, $0 \leq \theta \leq 2\pi$, and $0 \leq z \leq H$. The temperature field $T(r, \theta, z, t)$ was discretized on a structured cylindrical grid with resolutions $N_r = 220$, $N_{\theta} = 360$, $N_z = 64$ such that $T_{i,j,k} \approx T(r_i, \theta_j, z_k, t_n)$. The

nodal coordinates were generated as $r_i = R_i + i\Delta r$, $i=0, \dots, N_r-1$, $\theta_j = j\Delta\theta$, $j=0, \dots, N_\theta-1$, $z_k = k\Delta z$, $k=0, \dots, N_z-1$, with uniform spacings $\Delta r = \frac{R_o - R_i}{N_r - 1}$, $\Delta\theta = \frac{2\pi}{N_\theta}$, $\Delta z = \frac{H}{N_z - 1}$.

Periodicity was imposed in the circumferential direction to represent the full annulus:

$$T(r, 0, z, t) = T(r, 2\pi, z, t)$$

which is implemented numerically by wrap-around neighbors in the θ -index during the centered-difference evaluation of $\frac{\partial^2 T}{\partial \theta^2}$.³⁴

The 3-dimensional transient temperature field $T(r, \theta, z, t)$ was advanced using an explicit Forward-Time Centered-Space (FTCS) finite-difference scheme applied to the cylindrical Laplacian.³⁵ The explicit update at the interior nodes is written as

$$T_{i,j,k}^{n+1} = T_{i,j,k}^n + \alpha_{eff}^n \Delta t (\nabla^2 T)_{i,j,k}^n$$

where $(\nabla^2 T)_{i,j,k}^n$ is evaluated using second-order centered differences in r , θ , and z ,³⁶ consistent with the cylindrical Laplacian, and $\alpha_{eff}^n = \frac{k_{eff}^n}{\rho c_p^n}$ is the effective thermal diffusivity evaluated at the representative disc temperature $T_{mean}(t_n)$ where $k_{eff}^n = k(T_{mean})$ and $c_p^n = c_p(T_{mean})$.³⁷ Forced convection to air was imposed through convective boundary conditions on the inner bore ($r=R_i$), outer rim ($r=R_o$), and both disc faces ($z=0$, $z=H$). Within the explicit FTCS Formulation, these convective conditions were implemented using a ghost-cell formulation, which defines virtual temperatures just outside the domain such that the centered-difference Laplacian stencil can be applied at boundary nodes while enforcing the convective heat flux.

Perforation-enhanced cooling was incorporated without explicitly meshing individual perforations by coupling diffusion with an internal convective sink term via Strang Splitting. At each global time step, Δt , the update follows the symmetric operator-split sequence:³⁸

1) Internal sink half-step over $\Delta t/2$ applied analytically as $T < -T_\infty + (T - T_\infty)e^{(-\frac{\beta(t)\Delta t}{2})}$ where $\beta(t) = \frac{h(t)A_{perforations}}{\rho c_p^n V_{solid}}$.

2) Diffusion full-step over Δt using FTCS with centered spatial differences in (r, θ, z) and periodicity in θ .

3) Internal sink half-step over $\Delta t/2$, again applied using the same exponential update.

To ensure numerical stability of the explicit diffusion update at the chosen spatial resolution, the diffusion full-step is performed with internal sub-cycling: each global step Δt is subdivided into n_{sub} smaller diffusion steps $\Delta t_{sub} = \Delta t/n_{sub}$ that satisfies a conservative FTCS Stability bound.

At each time step, the temperature-dependent properties are updated consistently by evaluating $c_p(T)$ and $k(T)$ from the prescribed piecewise-linear tables at the volume-weighted mean disc temperature $T_{mean}(t^n)$, then computing $c_p^n = c_p(T_{mean})$, $k_{eff}^n = k(T_{mean})$, and $\alpha_{eff}^n = \frac{k_{eff}^n}{\rho c_p^n}$.³⁹ The time-varying convection coefficient $h(t)$ is obtained from the prescribed velocity series via the power-law model, and effective edge coefficients $h_{in}(t)$ and $h_{out}(t)$ are computed when perforation edge-enhancement is applied.

Outputs:

For each simulation case, the solver produces a single multi-page PDF containing absolute mid-plane temperature contours at 1 s intervals from $t=0$ to $t=15$ seconds. The mid-plane is defined at $z=H/2$, because $N_z=64$ is even, the true mid-thickness plane lies between the 2 central z -slices, thus the plotted mid-plane field is taken to be the average of the 2 central slices. Each of the 2 PDF outputs is a plane-view of the annulus (a carbon-carbon composite brake disc with 800 and 1000 perforation jacket) on the x - y plane, with the inner and outer radii overlaid, and includes a colorbar in Kelvin. The page title reports time t , the predetermined velocity $v(t)$, the base face convection coefficient $h(t)$, and the case identifier.

At each global time step Δt , the update proceeds as an Internal sink half-step over $\Delta 0.5t$, diffusion full-step over.

Results and Discussion

1D Baseline: Variable-conductivity rod with internal heat source:

A one-dimensional rod model was implemented as a baseline case to isolate conductive heat redistribution and end-losses under convection. The domain was a rod of length $L=1$ m, discretized using $N_x=50$ uniformly spaced nodes. The material properties were taken as representative carbon-carbon composite values and implemented in the transient heat equation with convection at both ends:

$$k(0)\frac{\partial T}{\partial x}(0, t) = h(T(0, t) - T_\infty), \quad -k(L)\frac{\partial T}{\partial x}(L, t) = h(T(L, t) - T_\infty)$$

where T_∞ is the ambient temperature, and h is the heat-transfer coefficient. Numerically, the partial differential equation was solved using an explicit FTCS finite-difference scheme, with a flux-conservative spatial discretization and convective boundary conditions enforced using a ghost-node formulation such that the boundary heat flux responded to $(T - T_\infty)$ rather than fixing the boundary temperature.

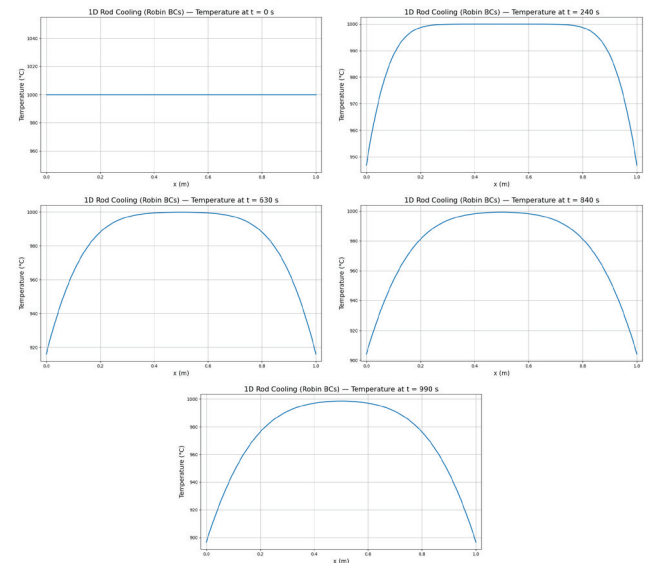


Figure 1: 1D carbon-carbon composite rod temperature profile $T(x,t)$ under convective boundary conditions at both ends, showing the spatial distribution after the reported simulation time. The rod cools towards the ambient temperature through boundary heat exchange governed by h .

Figure 1 shows a cooling profile consistent with a symmetric setup (uniform initial temperature of $1000\text{ }^{\circ}\text{C}$ and identical convection at both ends). The temperature remains the greatest in the center while the ends cool more rapidly due to the convective boundaries permitting the loss of heat from the domain in proportion to $(T-T_{\infty})$. As time increases, the thermal gradient near each boundary steepens, then gradually decreases as the entire rod approaches T_{∞} ($23\text{ }^{\circ}\text{C}$). This baseline case ensures that the numerical discretization and Robin boundary implementation produce physically expected behavior.

2D temperature-field evolution under convective (Robin) cooling:

A carbon-carbon composite square plate of side length 0.5 m was simulated using the transient two-dimensional heat equation with convective (Robin) boundary conditions applied to exposed boundaries. Across all configurations, temperature fields were depicted at regular time intervals, simulating the decrease in the temperature of the plate. The investigated geometries were: (1) a plate with no perforations, (2) a plate with a singular, central 10 cm diameter perforation at the center of the plate, (3) a plate with 25 regularly arranged, evenly divided perforations with a diameter of 1 cm , (4) a plate with 16 regularly arranged, evenly divided perforations with a diameter of 1 cm .

Solid carbon-carbon composite plate (with the outer edges having convective boundary conditions):

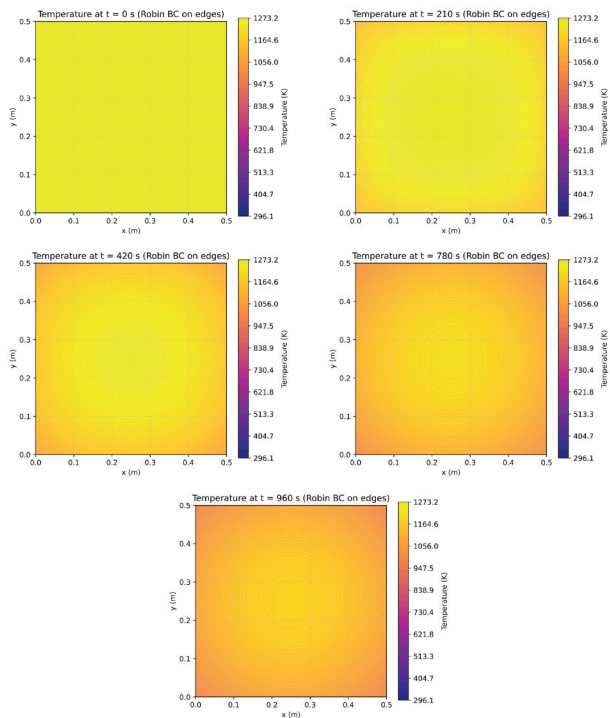


Figure 2: Solid carbon-carbon composite plate simulation with temperature maps at 30 s intervals from 0–960 s with convective cooling on all outer edges.

This baseline result, depicted in Figure 2, shows the expected diffusion-dominated behavior: the strongest gradients develop at the external boundaries where convective heat loss occurs, while the interior remains hotter for longer because heat must

be conducted to the perimeter before it can be removed. As time increases, the temperature field becomes progressively smoother as diffusion reduces gradients throughout the domain, and the solution approaches a low-gradient state consistent with sustained convective cooling at the edges.

Carbon-carbon composite plate (with a singular central 10 cm diameter circular perforation at the center of the plate)

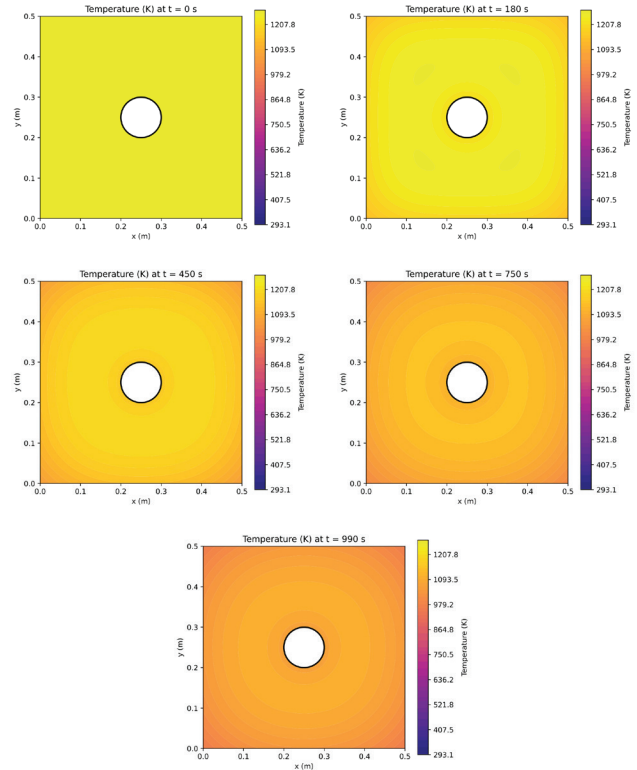


Figure 3: Single-holed (single perforation) carbon-carbon composite plate simulation (with the outer edges and the inner perforation-fluid boundary having convective boundary conditions).

Introducing a single circular perforation at the center of the plate adds an internal solid-fluid interface where convective boundary conditions are also applied (along with those at the outer edges of the plate). As such, the isotherms (lines on a graph indicating equal temperature) are not solely controlled by distance to the external perimeter; rather, the interior temperature structure is shaped by simultaneous conduction inwards towards the central perforation and conduction outwards towards the outer edges of the plate, as shown in Figure 3. However, as time increases, diffusion decreases, and the field becomes smoother.

Carbon-carbon composite plate (with 16 circular perforations with a 1 cm diameter arranged regularly):

This configuration reduces the characteristic conduction distance from interior points to a convective boundary, which promotes a more spatially distributed cooling pattern. Over long times, diffusion still smooths the field, but the approach to a low-gradient state is mediated by several heat-exchange locations evenly distributed across the plate, as depicted in Figure 4.

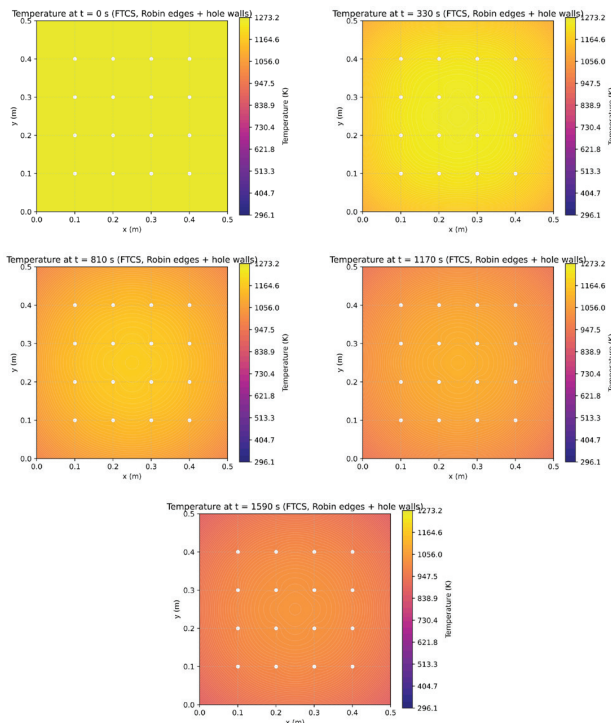


Figure 4: 16-holed(16-perforation) carbon-carbon composite plate simulation (with the outer edges and the 16 inner perforation-fluid boundaries having convective boundary conditions).

Carbon-carbon composite plate (with 25 circular perforations with a 1 cm diameter arranged regularly)

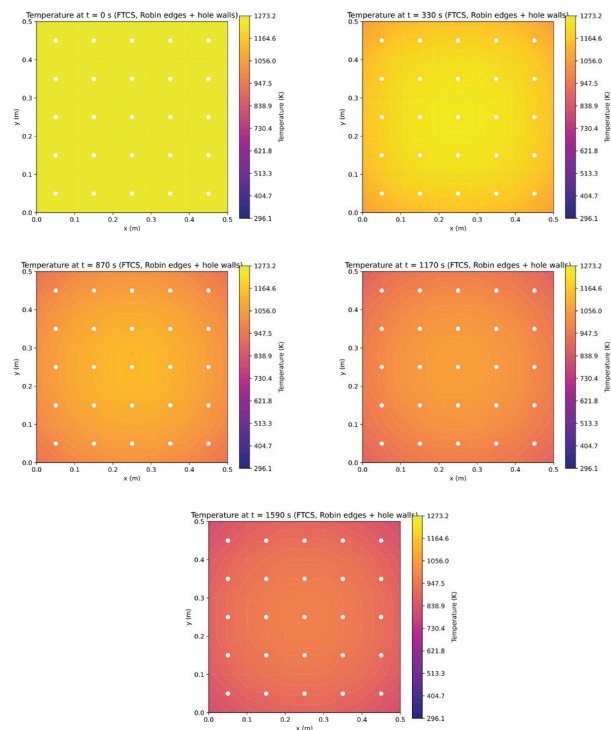


Figure 5: 25-holed (25-perforation) carbon-carbon composite plate simulation (with the outer edges and the 25 inner perforation-fluid boundaries having convective boundary conditions).

Compared to the plate with 16 perforations, a more even pattern of heat distribution as many local convective inter-

faces influence the temperature simultaneously. However, the presence of many (25) evenly distributed sinks also means that large interior regions are no longer insulated from convection by long conduction paths to the perimeter, as evidenced by Figure 5. Over time, diffusion homogenizes the field, but the trajectory toward a low-gradient, near-ambient state is driven by heat removal at numerous internal boundaries rather than predominantly at the external edges.

Physical application of this simulation:

Within this simplified model, the simulations demonstrate that perforations within the material can act as distributed thermal sinks, reshaping transient temperature patterns. This supports the design intuition that perforations do more than reduce mass: they also alter the thermal pathway topology by adding internal convective boundaries where heat loss can occur.

A 25-perforation configuration cools faster than a 16-perforation configuration as it introduces more thermally active internal boundaries and also reduces the distance from any point on the plate to a boundary (assuming the perforations are evenly distributed). This is evident in Figure 6 (below) when the orange line indicating 25 perforations has a smaller gradient than the blue line indicating 16 perforations, therefore showing that the greater the number of perforations, the higher the rate of cooling.

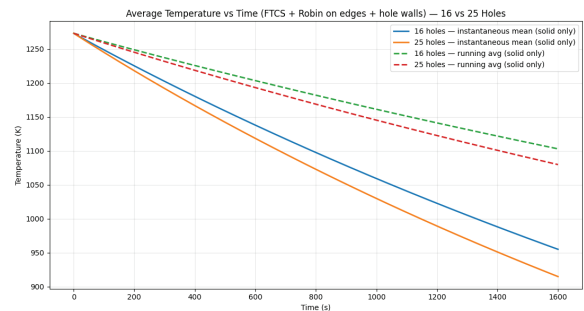


Figure 6: Combination of the graphs of Temperature against time for 16 and 25 (holes) perforations (Note: the x-axis is very elongated, thus it may be difficult to view the graph as nonlinear).

3D FTCS Brake disc cooling:

Two perforation cases were simulated for the Mercedes W05 brake disc under the same 15 s acceleration-cooling period, where the car reaches maximum speed at 14 s: 800 perforations and 1000 perforations. In both simulations, the disc was initialized at a uniform temperature T_0 and cooled toward ambient T_∞ through forced convection applied on the inner bore, outer rim, and both brake disc faces, with perforation-enhanced cooling represented by an internal volumetric sink and an edge convection boost.

Figure 7 and Figure 8 present the peak and volume-mean temperature histories predicted by the 3-dimensional FTCS Carbon-carbon composite brake disc model over 15 s during the simulated acceleration of the Mercedes W05 out of turn 2 in the Autodromo Nazionale Monza Circuit. For both the 800-perforation and 1000-perforation simulations, the peak temperature remains above the volume-mean temperature throughout the simulation, indicating that spatial non-uni-

formity persists as the disc cools. The non-linear curvature of the graphs is consistent with the imposed time-dependent forced-convection: As the prescribed airflow increases with time, the effective convective heat removal increases, accelerating the decay of both the peak and mean temperatures. This curvature of the 2 simulations reflects competing effects. Because $h(t)$ varies in time, while the driving temperature difference ($T-T_\infty$) decreases as the disc cools, weakening the instantaneous convective driving, with further modulation from $C_p(t)$ and $k(t)$.

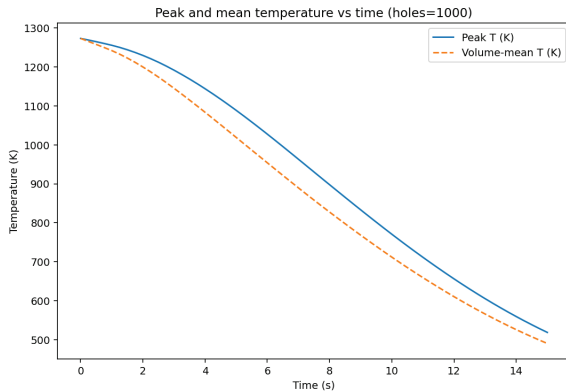


Figure 7: Peak and volume-mean disc temperature against time graph for the 1000-perforation (hole) annular disc case over 15 s under velocity-driven convection, in Kelvin.

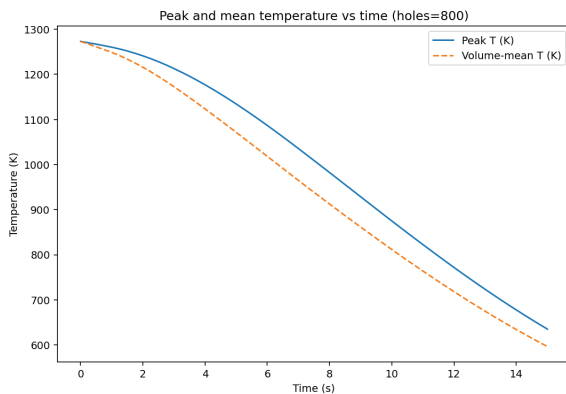


Figure 8: Peak and volume-mean disc temperature against time graph for the 800-perforation (hole) annular disc case over 15 s under velocity-driven convection, in Kelvin.

A clear geometric effect is observed when comparing the 2 graphs. The 1000-perforation case cools more rapidly than the 800-perforation case at all times t , with the difference in the temperature increasing near the end of the simulations as the cumulative effect of the stronger internal sink (of the 1000-perforation simulation) increases. By $t \approx 15$ s, the 1000-perforation simulation reaches approximately $T_{peak} \approx 5.2 \times 10^2 K$, whereas the 800-perforation case remains higher at approximately $T_{peak} \approx 6.3 \times 10^2 K$, a difference of approximately 100 K when comparing variables for each simulation. This difference arises from the increased effective internal convective area and associated volumetric sink strength in the higher perforation disc, due to the increased heat-transfer surface area, which increases the net heat loss from the solid brake disc without explicitly meshing individual perforations.

The separation between peak and mean temperature also provides a compact indicator of thermal gradients: where the gap is greater, the field is more heterogeneous, suggesting a greater likelihood of local hot-spot persistence even as the bulk temperature decreases.

From a practical standpoint, under identical airflow forcing and material property updates, the higher the number of perforations (and thus the higher the total surface area) of the brake disc, the higher the overall rate of cooling of the brake disc, decreasing the peak temperature over the 15 s acceleration (and thus cooling) time period, suppressing the hottest regions more effectively over this period of time, suggesting a greater thermal-management margin for the more heavily perforated brake disc. This is evident in Figure 9 (below).

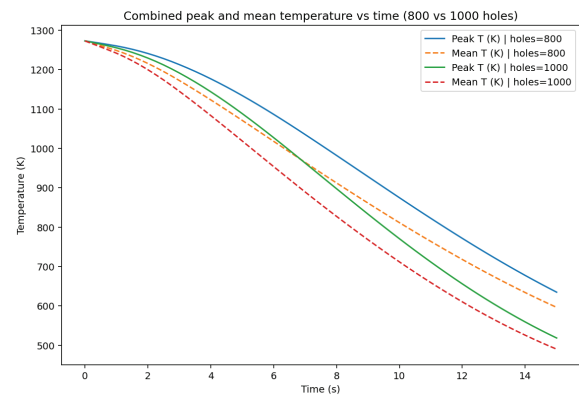


Figure 9: Combination of the graphs of Temperature against time for the 800-perforation and 1000-perforation simulations.

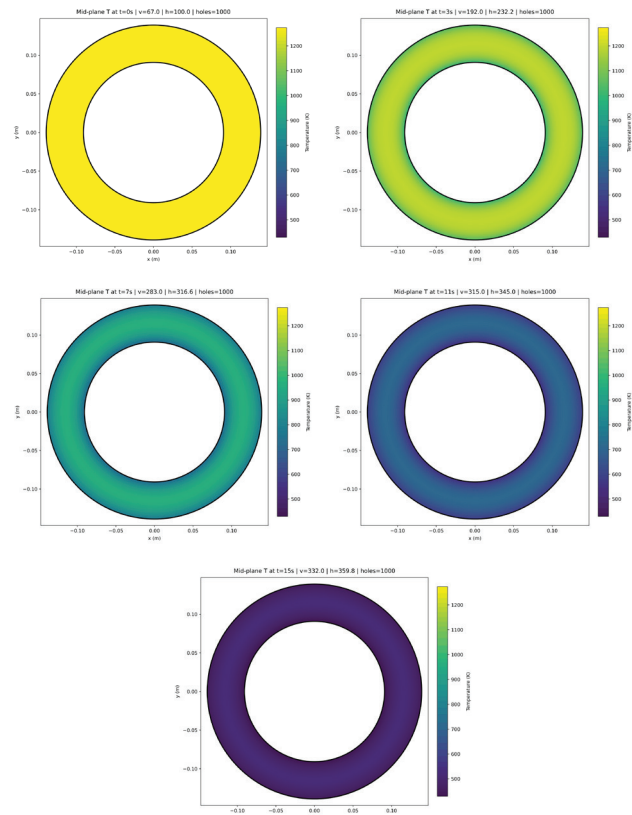


Figure 10: Mid-plane temperature contours for the 1000-perforation annular brake disc over 15 s under velocity-driven convection.

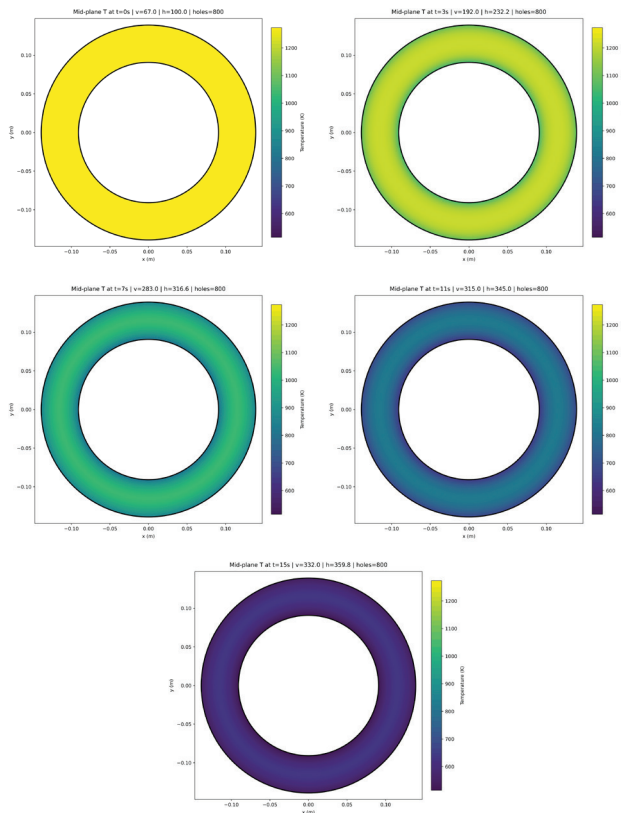


Figure 11: Mid-plane temperature contours for the 800-perforation annular brake disc over 15 s under velocity-driven convection.

Across both perforation configurations, the multi-page mid-plane temperature field outputs (figures 4 and 5) indicate the qualitative cooling topology. At time $t=0$ s, both cases are spatially uniform at the prescribed initial temperature, $T_0 = 1273.15$ K, with cooling first developing as radial boundary layers adjacent to the inner bore and outer rim before progressively penetrating toward mid-radius as time increases. This is consistent with the imposed convective boundary conditions at the edges of the brake disc and faces of the brake disc, and with the monotonic increase in the prescribed airflow-driven convection during the acceleration of the W05 F1 car out of turn 2 in Monza. Simultaneously (as time advances), the fields remain circumferentially uniform, which is expected because the boundary conditions are applied uniformly around the brake disc and periodicity is enforced in θ . The dominant spatial structure is thus radial, with the inner bore and outer rim cooling the fastest, producing cooler bands at $r=R_i$ and $r=R_o$, while the interior retains higher temperature at early times. This behavior is consistent with convection acting on all exposed boundaries, with an increasing forced convection intensity through the prescribed $h(t)$ during the 0–15 s acceleration period.

When the 2 simulations are compared directly, the 1000-perforation configuration cools more rapidly than the 800-perforation configuration under the same external airflow, at the same time. The physical reason is geometric: increasing the number of through-radial perforations increases the total internal convective surface area used by the distributed sink term and also strengthens the effective edge convection

through the edge boost allocation, so the disc experiences a larger net heat removal capacity for the same external airflow history. This difference is also visible in the mid-plane contours, where the 1000-perforation simulation exhibits an earlier and more extensive shift towards cooler color levels at the same time compared to the 800-perforation simulation. Interpreted physically within the model assumptions, increasing perforation count increases the number of thermally active internal boundaries, via the sink term, and enhances edge convection, reducing both the absolute temperature level and the persistence of hot regions during rapid post-braking cooling.

Model limitations:

1) Thermal radiation is not modeled in the simulation:

Adding a realistic radiation model to my simulation involves not just introducing T^4 on the surface. In practice, radiation can require nonlinear boundary terms, temperature-dependent emissivity, and coupling with convection and conduction at every step. With the already demanding FTCS 3D grid, adding radiative exchange would substantially increase the runtime and memory usage and therefore would require much more powerful computing devices than what is currently available. In fact, major companies such as AWS and Oracle partner with F1 teams, providing services for computational analysis of their Formula 1 cars.

Overall, however, with the current high-resolution 3D explicit FTCS framework, incorporating coupled enclosure radiation would substantially increase computational cost and implementation complexity, often motivating the use of high-performance computing resources for iterative design-grade studies.

Additionally, due to the compact nature of the modern drum-shaped F1 brake system, any heat radiated from the brake disc will be absorbed by this dark colored carbon-fiber drum and will be re-radiated back towards the brake disc. As such, there should not be much variation between the current simulation employed and the simulation updated with a radiation model.

For future work, one may consider non-linearities such as the Stefan-Boltzmann Law, which accounts for thermal radiation.

2) The convection coefficient $h(t)$ is a simplified surrogate:

The convection coefficient is prescribed through a power-law dependence on a velocity trace. This captures the trend of increasing forced convection during acceleration, but it does not fully resolve full under-wheel aerodynamics, brake duct geometry, or turbulence structure.

This simplified convection model is employed because realistic forced convection around an F1 brake disc is strongly governed by 3-dimensional unsteady flow: Rotating wheel pumping, wheel-wake interaction, brake-disc jet impingement, strong curvature and separation, and turbulence. In a higher-resolution, higher-fidelity approach, the local heat transfer coefficient h is not prescribed; it emerges from a conjugate heat transfer computation where the airflow and disc conduction are solved in a coupled manner, producing a spatially and temporally varying all heat flux. Achieving this fidelity typically demands fine near-wall meshing, turbulence modeling, and

small time steps, leading to large cell counts and long runtimes, a simulation that would require the use of powerful computational devices.

Therefore, h is prescribed in my model as an engineering surrogate to capture the first-order effect of increasing forced convection during acceleration while keeping the model computationally tractable.

3) Perforations in the brake disc are not explicitly meshed:

Approximating the perforations as a distributed sink and an edge boost avoids meshing thousands of features, yet it fails to capture local jetting and recirculation.

The unmeshed model in the simulation is used, since meshing thousands of perforations substantially increases the grid size and runtime. To resolve a perforation with diameter several cells across it in each direction are necessary. Repeating this process for up to 1000 perforations increases the grid into hundreds of millions of cells, which is unfeasible without the use of powerful computing devices.

4) Effective isotropic conductivity and representative temperature:

In this model, the temperature-dependent thermal conductivity $k(T)$ and specific heat $c_p(T)$ are implemented as piecewise-linear tables, but (for computational tractability in the High-resolution 3-dimensional explicit FTCS scheme) they are evaluated once per time step using the volume-weighted mean temperature $T_{mean}(t)$. This gives stepwise-constant effective values $k_{eff} = k(T_{mean})$ and $c_{p,eff} = c_p(T_{mean})$ that are applied uniformly across the domain for that time step. However, this model forces the whole disc to use the same k_{eff} and $c_{p,eff}$ based on T_{mean} , so hotspots and cooler regions cannot conduct or store heat differently, despite $k(T)$ and $c_p(T)$ are temperature-dependent. Consequently, the model can misestimate local cooling rates and temperature gradients, reducing accuracy for predicting hotspot persistence and subtle small-scale thermal non-uniformity.

The model employs this simplified material behavior because updating $k(T)$ and $c_p(T)$ everywhere would be computationally expensive. The simulation evaluates properties at the volume-weighted mean temperature, once per global step, and uses that for k_{eff} , $c_{p,eff}$ and α_{eff} . This prevents the necessity of recalculating variable coefficients at every node sub-step, which would significantly raise the computational cost whilst still maintaining a relatively high degree of accuracy.

Physical implications of this simulation:

Within this simplified FTCS-based 3-dimensional model, a comparison between the 1000-perforation and 800-perforation results from this simulation indicates a consistent acceleration of the rate of cooling when the number of perforations is increased, evidenced both by the peak temperature and volume-mean temperature plots in Figure 7 and Figure 8. In both simulations, the curves are monotonically decreasing over the 0-15 second window, but the 1000-perforation case remains systematically lower after the initial transient, indicating that the additional perforation area increases the resultant

heat-removal rate through the combined mechanisms used in the model.

By the end of the simulated interval, the 1000-perforation case reaches a peak temperature of approximately 520 K and a volume-mean temperature of approximately 500 K, whereas the 800-perforation case remains higher at roughly 630 K and a volume-mean temperature of 600 K. This corresponds to an end-of-window reduction on the order of $\Delta T \approx 110$ K in both peak and mean when moving from 800 to 1000 perforations.

In a 2014 Formula 1 operating context, that scale of reduction is sufficiently large to be meaningful: it implies less time spent at elevated brake disc temperatures during the corner-exit acceleration phase, which decreases the probability of brake overheating and thus brake failure, thus increasing repeated braking performance, and reduces the possibility of brake overheating (despite the high maximum temperature which can be withstood by carbon-carbon composite brakes) and brake failure.

■ Conclusion

This study developed a progressively higher-fidelity transient heat-conduction framework for motorsport brake cooling. Beginning with 1-Dimensional and 2-Dimensional FTCS finite-difference simulations with convective (Robin) boundary conditions to confirm that the explicit FTCS diffusion scheme behaves correctly via the 1D case. It also ensured that the scheme is applicable to a larger field with geometric effects, before simulating the computationally expensive 3-Dimensional simulation. In the 3D simulation, the carbon-carbon composite disc representative of the 2014 Mercedes W05 Formula 1 racecar was initialized uniformly at $T_0 = 1273.15$ K and cooled over 15 s, towards ambient air $T_\infty = 296.15$ K using time-dependent convective boundary conditions on the inner bore, outer rim and both faces of the brake disc, where increasing convection coefficient $h(t)$ is driven by the prescribed, increasing velocity history; the mid-plane fields and peak volume-mean graphs depict boundary-initiated cooling with persistent internal non-uniformity over this period of time. Under identical material parameters and conditions, the 1000-perforation configuration cools at a higher rate and reaches lower peak and mean temperatures compared to the 800-perforation configuration, as the modeled effective convective area (the total surface area) of the brake disc is greater due to the presence of an additional 200 perforations for the 1000-perforation configuration simulation (compared to the 800-perforation configuration simulation). Therefore, according to the model employed, it follows that the greater the surface area in contact with the surrounding fluid, the greater the rate of cooling. In motorsport, namely, Endurance racing, Indycar and Formula 1, where brake thermal margins are narrow, as engineers are persistently working to shave hundredths of a second off of lap times, and because consistent high performance braking is repetitive, these simulations are significant as improved heat loss rate can stabilize brake response when

there are frequent heavy braking zones and reduce the risk of thermally driven performance loss across each lap.

■ Acknowledgments

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