

Plasma Braking in Subcritical X-ray Pulsars: Coulomb Deceleration in Strongly Magnetized Atmospheres

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ABSTRACT: We investigate plasma braking in sub-critical X-ray pulsars (XRP), where radiation pressure is negligible and infalling protons are decelerated primarily by Coulomb collisions in a strongly magnetized neutron-star atmosphere. Our key advance is a self-consistent numerical model that couples the evolution of the flow velocity and the atmospheric density, rather than prescribing a fixed background atmosphere or an ad hoc stopping depth. The model integrates the coupled momentum and deceleration equations while evaluating a generalized Coulomb logarithm applicable across classical, quantum, and strongly magnetized regimes, incorporating the Debye length, quantum diffraction, and electron gyroradius (Larmor radius). We compute stopping lengths and compression profiles across a range of magnetic field strengths, temperatures, densities, and compositions. We find that self-consistent atmospheric compression steepens the upper-layer density gradient and can significantly modify the depth and shape of the braking region compared to fixed-density estimates. These vertical structure profiles have direct implications for radiative transfer and vacuum-resonance mode conversion, providing physically grounded inputs for spectral and polarization modeling of accreting neutron stars.

KEYWORDS: Physics and Astronomy, Astrophysics, Neutron Stars, X-Ray Pulsars, Plasma, Atmospheres.

■ Introduction

Accretion of matter onto a strongly magnetized neutron star (NS) gives rise to X-ray pulsars (XRP).¹ In XRP, the surface magnetic field strength is confirmed to be at least $\geq 10^{12}$ G, which drastically alters both the accretion flow geometry and the microphysics of radiation-matter interaction.² The luminosities of XRP span an enormous range, from about 10^{33} erg s⁻¹ up to $\sim 10^{41}$ erg s⁻¹. At the low end, accretion either halts due to the transition into the “propeller” regime³⁻⁵ or proceeds in a stable fashion from a recombined cold disc.⁶ At the highest luminosities, some XRP are classified as ultra-luminous X-ray sources.^{7,8} The strong magnetic field channels matter onto small regions near the magnetic poles of the NS, where most of the gravitational energy is released. The structure of this emission region is determined by the mass accretion rate and corresponding luminosity. At relatively low luminosities ($L \lesssim 10^{37}$ erg s⁻¹), the accretion flow is stopped in the NS atmosphere, and radiation escapes from hot spots of area $\sim 10^9$ cm².^{9,10} When $L \gtrsim 10^{37}$ erg s⁻¹, the deceleration occurs above the surface in a radiation-dominated shock, producing extended accretion columns.¹¹⁻¹³ Spectra of XRP are usually modeled with a power law plus a high-energy cutoff. At very low accretion rates ($L \lesssim 10^{35}$ erg s⁻¹), some sources display a spectral transition and develop a two-component spectrum.¹⁴⁻¹⁶ Although models exist for both the low-luminosity regime^{17,18} and standard luminosity states,¹⁹ no single theory yet explains the spectral transitions self-consistently. Independent of the model details, the main radiative processes include magnetic Compton scattering,²⁰ free-free emission and absorption,²¹ and cyclotron processes.²² These processes depend strongly on the polarization state of X-ray photons, especially below

the electron cyclotron energy $E_{\text{cyc}} \simeq 11.6 (B/10^{12} \text{ G}) \text{ keV}$. Because the opacity is polarization-dependent, high linear polarization degrees ($\sim 80\%$) have been predicted by several XRP models.^{18,23-25} However, observations with IXPE26 show systematically lower polarization levels for all XRP studied so far.²⁷⁻³³ One factor influencing the emergent polarization is the conversion of polarization modes near the vacuum resonance.^{21,34} In a strongly magnetized plasma, radiative transfer is described in terms of two normal modes whose polarization is set by both plasma and vacuum contributions to the dielectric tensor.³⁵⁻³⁸ In the dense lower atmosphere, plasma effects dominate, while in the tenuous upper layers, vacuum polarization becomes important. As the modes propagate outward, their polarization vectors can interchange, depending on the local mass density gradient: for shallow gradients, nearly adiabatic O-X or X-O conversion occurs, while steep gradients lead to partial or non-adiabatic mode conversion.²¹ A key factor shaping the mass density profile in XRP is the braking of the accretion flow as it penetrates the atmosphere. The momentum transfer from protons to the atmospheric plasma compresses the upper layers and modifies the density gradient, thereby affecting both radiative transfer and polarization formation. In this work, we focus specifically on the physics of plasma braking in sub-critical XRP, where the radiative force is negligible, and Coulomb collisions dominate the deceleration process. We investigate how the flow-atmosphere interaction sets the vertical structure and discuss its implications for X-ray emission and polarization. Previous work has shown that Coulomb interactions can efficiently brake the accretion flow in the low-luminosity regime and that the resulting density gradient is important for radiative transfer and polarization formation.

However, many treatments effectively assume a background atmospheric structure (or prescribe a stopping depth) and then compute deceleration on top of it. In reality, momentum deposition from the decelerating flow compresses the atmosphere, and that compression in turn feeds back on the Coulomb drag through the local density.

The key contribution of this paper is a self-consistent numerical model that couples the evolution of the flow velocity and atmospheric density in sub-critical X-ray pulsars. Unlike previous approaches that prescribe a fixed background atmosphere or stopping depth, we simultaneously integrate the coupled momentum and deceleration equations, incorporating a generalized Coulomb logarithm valid across classical, quantum, and strongly magnetized regimes. This approach enables us to predict stopping lengths and compression profiles without imposing them by hand. Our results show that self-consistent atmospheric compression steepens the upper-layer density gradient and significantly modifies both the depth and shape of the braking region compared to fixed-density estimates. We find that stopping lengths range from ~ 0.01 cm in cool, high- Z atmospheres to ≥ 1 cm in hot, light compositions, with most kinetic energy dissipated in the bottom half of the braking layer where density and Coulomb coupling are largest. This coupling allows us to predict the stopping length and the braking-induced density gradient without imposing them by hand, yielding physically motivated vertical profiles that can be used directly in spectral and polarimetric calculations.

■ Theoretical model

Structure of Neutron Star Atmosphere.

The vertical structure of a neutron-star atmosphere in a subcritical X-ray pulsar is governed by the local gravitational field, the thermal pressure of the plasma, and the momentum exchange between the infalling accretion flow and the atmospheric layers. In this luminosity regime, radiation pressure is negligible, and the deceleration of the accretion flow is dominated by Coulomb collisions between the infalling protons and the atmospheric electrons. In several previous studies, the effect of plasma braking has been incorporated through a prescribed momentum-deposition profile, often parameterized as an exponential decay with optical depth, which provides a convenient phenomenological description of the stopping layer. While such prescriptions capture the localization of momentum transfer, they do not account for the feedback between flow deceleration and atmospheric compression, and therefore cannot predict the braking depth or density gradient self-consistently. In this work, we do not assume a predefined braking profile. Instead, we compute the deceleration of the accretion flow directly from Coulomb interactions, using a generalized Coulomb logarithm that remains valid in classical, quantum, and strongly magnetized regimes. The resulting braking rate depends explicitly on the local plasma density and temperature, which evolve as the atmosphere responds to momentum deposition. As a consequence, the velocity of the inflow and the atmospheric density structure are intrinsically coupled and must be solved simultaneously.

Under the assumption that the atmosphere adjusts quasi-statically to the applied force (justified by the short sound-crossing time compared to the accretion timescale), the atmospheric structure can be treated as being in local hydrostatic equilibrium, with an additional force term arising from momentum transfer by the decelerating flow. The vertical momentum balance can be written as

$$\frac{dP}{dz} = -\rho g + \frac{\dot{M}}{S} \frac{dv}{dz} \quad (1)$$

where the second term on the right-hand side represents the momentum transfer from the decelerating accretion flow to the atmospheric plasma. Here the gas pressure is given by $P = \rho kT/(\mu m_p)$, ρ is the local mass density, g is the gravitational acceleration, T is the local temperature, μ is the mean atomic weight, m_p is the mass of a proton, S is the area of the hot spots at the NS surface, v is the local velocity of the accretion flow, and $\dot{M}$ is the mass accretion rate. The deceleration law for the flow is determined by Coulomb interactions between infalling protons and electrons in the atmosphere:

$$\frac{d}{dz} \left(\frac{m_p v^2}{z} \right) = - \frac{4\pi e^4 Z^2 n_e}{m_e v^2} \ln \Lambda \quad (2)$$

where $\ln \Lambda$ is the Coulomb logarithm. While equation (2) is written for a non-magnetized plasma, it remains applicable in the magnetized case if $\ln \Lambda$ is replaced by the appropriate magnetized expression.³⁹ This approach allows the effects of strong magnetic fields to be incorporated through the microphysics of Coulomb scattering without modifying the macroscopic form of the braking equation. Introducing the dimensionless velocity $\beta = v/c$, the equation becomes:

$$\frac{d\beta}{dz} = 3.24 \times 10^{-4} \frac{\rho z^2}{\beta^3} \ln \Lambda \quad (3)$$

Combining equation (3) with the momentum balance equation (1), we obtain the density gradient in the atmosphere:

$$\frac{d\rho}{dz} = 0.1 \frac{\mu g_{14} \rho}{T_{\text{keV}}} + 31.15 \frac{\mu \dot{m}_6}{T_{\text{keV}}} \frac{d\beta}{dz} + \frac{\rho}{T_{\text{keV}}} \frac{dT_{\text{keV}}}{dz} \quad (4)$$

where $g_{14} \simeq 1.8 M_{1.4} R_6^{-2}$ is the gravitational acceleration in units of $10^{14} \text{ cm s}^{-2}$, $M_{1.4}$ is the NS mass in units of solar masses, R_6 is the NS radius in units of 10^6 cm , T_{keV} is the temperature in units of keV, and

$$\dot{m}_6 \simeq 25 \dot{M}_{17}^{3/5} B_{12}^{1/2} \quad (5)$$

is the mass accretion rate per unit area measured in $10^6 \text{ g s}^{-1} \text{ cm}^{-2}$, $\dot{M}_{17}$ is the mass accretion rate in units of 10^{17} g/s , and B_{12} is the surface magnetic field strength in units of 10^{12} G . In equation (4), the first term reflects hydrostatic compression by gravity, the second term represents additional atmospheric compression caused by momentum deposition from plasma braking, and the third term accounts for temperature gradients. In the absence of accretion ($\dot{m}_6 = 0$), equation (4) reduces to the standard hydrostatic equilibrium condition for a plane-parallel atmosphere. The density at the top of the atmosphere is set equal to the density of the accretion stream:

$$\rho_a = \frac{\dot{M}}{S_{\text{eff}}} \approx 2.5 \times 10^{-4} \frac{\dot{M}_{17} R_6^{1/2}}{S_{10} M_{1.4}^{1/2}} \text{ g cm}^{-3} \quad (6)$$

which serves as the upper boundary condition for integrating equation (4). This condition assumes a smooth matching between the accretion stream and the atmospheric plasma; any finite transition layer would primarily affect the very top of the atmosphere and does not significantly alter the stopping depth or compression profile derived below.

The current model is designed for subcritical accretion onto strongly magnetized neutron stars. In this scenario, radiation pressure is minor compared to gravity, and Coulomb collisions primarily slow down the incoming plasma. The atmosphere is treated as plane-parallel and in steady state. This approach works well because the polar-cap footprint is small, and the sound-crossing time is short compared to the timescale of accretion. However, the model does not include radiative energy transport or temperature feedback in a self-consistent way; instead, it fixes the temperature profile to focus on the effects of plasma braking and atmospheric compression. Other effects are also ignored, in large part due to their relative insignificance. For example, multidimensional effects are neglected, as at the level of Coulomb collisions, the surrounding plasma remains almost perfectly uniform; likewise, magnetic field curvature is essentially negligible at such scales. General relativistic corrections are not taken into account, as these do not fundamentally change the electromagnetic interaction between particles. Given these assumptions, the model effectively shows the key relationship between Coulomb deceleration and atmospheric structure in the plasma-braking regime.

Coulomb Logarithm in Strong Magnetic Field and Low-Density Regime:

The Coulomb logarithm is generally defined as:

$$\ln \Lambda = \ln \left(\frac{b_{\max}}{b_{\min}} \right) \quad (7)$$

where $b_{\max}$ is the maximum impact parameter (typically the Debye length λ_D), and $b_{\min}$ is the minimum impact parameter, which requires more careful treatment in low-density, strongly magnetized, or quantum plasmas.

Thus, in this work, we use a generalized expression for the Coulomb logarithm, $\ln \Lambda$, that remains valid in the presence of strong magnetic fields and extends to low-density plasma, where traditional approximations may break down. We adopt a synthesized formula that combines three physical scales: the classical distance of closest approach, the de Broglie wavelength, and the electron gyroradius (or Larmor radius). While no single prior publication has employed a similar generalization, the individual parameters are commonly seen. The classical distance of closest approach appears in non-relativistic Coulomb collision theory, while the de Broglie wavelength is used as the quantum lower bound on spatial resolution, and the Larmor radius limits perpendicular scattering in magnetized plasmas. We adopt the maximum of these three scales as $b_{\min}$ because at any given point, the collision process is regulated by whichever constraint is tightest, and taking the maximum ensures we do not underestimate the effective cutoff. The resulting formula provides a continuous and phys-

ically meaningful $\ln \Lambda$ across the full parameter space explored in this work.

$$\ln \Lambda = \ln \left(\frac{\lambda_D}{\max \left\{ \frac{e^2}{kT}, \frac{\hbar}{\sqrt{2m_e kT}}, \frac{\sqrt{2kT/m_e}}{\omega_{ce}} \right\}} \right) \quad (8)$$

where $\lambda_D = \sqrt{\frac{kT}{4\pi e^2 n_e}}$ is the Debye length, $\frac{e^2}{kT}$ is the classical distance of closest approach (i.e., the closest classical approach distance in the absence of quantum effects),⁴⁰ $\frac{\hbar}{\sqrt{2m_e kT}}$ is the de Broglie wavelength (this acts as the lower limit on spatial resolution due to wave-particle duality),⁴¹ $\frac{\sqrt{2kT/m_e}}{\omega_{ce}}$ is the electron Larmor radius (in strongly magnetized plasma the motion perpendicular to the field becomes quantized, limiting the effective minimum distance for collisions),⁴² $\omega_{ce} = eB/m_e c$ is the electron cyclotron frequency. Expression (8) ensures a physically meaningful $\ln \Lambda$ even when the plasma is near the collisionless regime or when quantum or magnetic effects dominate. For applications in neutron star envelopes and magnetized accretion flows, this form is consistent with previous works,⁴³⁻⁴⁵ where magnetic field and quantum effects are included in transport coefficient calculations. This generalized expression is especially useful in simulations where plasma parameters evolve across regimes and a uniform treatment of $\ln \Lambda$ is desirable.

Throughout all models, the Coulomb logarithm $\ln \Lambda$ remains in the physically meaningful range $5 \lesssim \ln \Lambda \lesssim 20$, depending on the local density, temperature, and degree of magnetization. In the upper atmosphere, where the electron gyroradius exceeds both the classical and quantum scales, $b_{\min}$ is set by the Larmor radius, making the collisions effectively magnetized. Deeper in the atmosphere, as the density increases, the Debye length contracts and the plasma transitions to the classical regime in which e^2/kT dominates $b_{\min}$. This evolution is naturally captured by the generalized expression of $\ln \Lambda$ and is reflected in the gradual variation of the braking efficiency with depth.

Numerical Model:

To model the deceleration of the accretion flow in the subcritical regime, we numerically integrate the coupled system of equations (3) and (4) governing the vertical evolution of the dimensionless velocity $\beta(z) = v/c$ and the local mass density $\rho(z)$. This approach simultaneously accounts for hydrostatic compression, plasma braking due to Coulomb collisions, and any temperature gradient effects. The Coulomb logarithm $\ln \Lambda$ appearing in equation (3) is evaluated self-consistently at each spatial step using the generalized expression given in equation (8). This form incorporates:

- 1) the classical distance of closest approach e^2/kT
- 2) the electron de Broglie wavelength $\frac{\hbar}{\sqrt{2m_e kT}}$, and
- 3) the electron gyroradius $v_\perp/\omega_{ce}$,

ensuring physical validity in classical, quantum, and strongly magnetized regimes. The maximum of these three scales is taken as $b_{\min}$, while the Debye length λ_D is used as $b_{\max}$, giving $\ln \Lambda = \ln(\lambda_D/b_{\min})$. The numerical domain begins at the top of the atmosphere, where the inflow velocity is set to β_i (typically corresponding to the local free-fall speed), and the density is fixed to the accretion stream value ρ_a from equation (6). Instead of prescribing a single value of μ , we specify the composition of the atmospheric plasma through the ion mass number A

and charge Z . The corresponding mean molecular weights are computed as $\mu = \frac{A}{Z+1}$, $\mu_e = \frac{A}{Z}$, representing the mean mass per particle (ions + electrons) and per electron, respectively. These quantities enter the pressure term and the Coulomb logarithm in equations (3) and (4). While the temperature can be held constant for simplicity, the formulation allows for an arbitrary $T(z)$ profile, in which case the third term in equation (4) is non-zero. We employ an explicit finite-difference scheme in the physical height x with a fixed step Δx small enough to resolve the steepest expected gradients. At each step:

- 1) Velocity update — We compute $\frac{d\beta}{dz}$ from equation (3) using the current ρ and $\ln\Lambda$, and update β via an Euler step.
- 2) Density update — With the new β , we then calculate $\frac{d\rho}{dz}$ from equation (4), which includes contributions from gravitational compression, momentum exchange from braking, and any imposed temperature gradient. The new density is then updated to the next step.

- 3) Termination — The integration stops when $\beta \rightarrow 0$ or becomes negative, which we define as the total stopping length, x_{stop} .

The output includes the full spatial profiles $\beta(x)$ and $\rho(x)$, as well as diagnostic quantities such as the local braking rate, the evolution of $\ln\Lambda$, and the fraction of kinetic energy dissipated at each depth. To highlight the influence of composition, we present results for different choices of (A, Z) , including hydrogen and carbon atmospheres, which correspond to $\mu = 1/2$ and $\mu = 12/7$, respectively. For clarity, we also show constant- ρ runs (to isolate kinematic effects) alongside full coupled β - ρ integrations (to capture the interplay between flow deceleration and atmospheric compression).

Numerical Results

In this section, we present the results of the coupled integration of the velocity and density structure of the neutron-star atmosphere for a range of temperatures and chemical compositions. Unless stated otherwise, all models assume a surface magnetic field $B = 10^{12}$ G, gravitational acceleration $g_{14} = 2.2$, initial velocity $\beta_i = 0.5$, and initial mass density $\rho_0 = 10 \text{ g cm}^{-3}$. The numerical step was fixed at $\Delta x = 10^{-3}$ cm, sufficient to resolve the shortest braking lengths obtained in any model.

Temperature Influencing Plasma Brake:

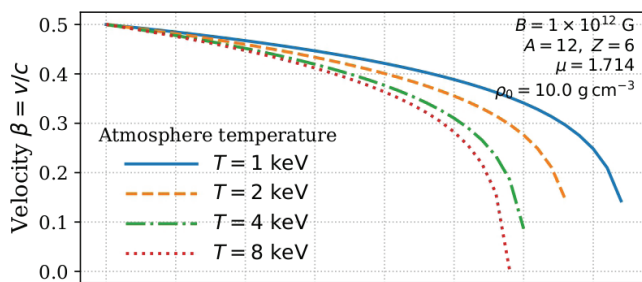


Figure 1: Velocity $\beta(x)$ profiles for a temperature scan of a fully ionized carbon atmosphere ($A = 12$, $Z = 6$, $\mu = 12/7$) with evolving density. Assume $B = 10^{12}$ G, surface gravity $g = 2.2 \times 10^{14} \text{ cm s}^{-2}$, initial density $\rho_0 = 10 \text{ g cm}^{-3}$, and initial velocity $\beta_i = 0.5$.

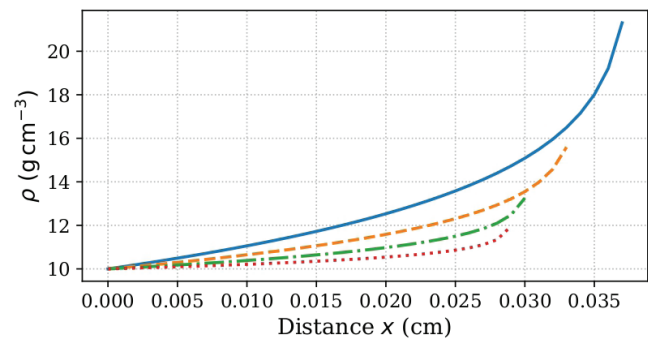


Figure 2: Density $\rho(x)$ profiles with the same initial parameters. The curves show solutions of the plasma-braking equations for constant temperatures $T = 1, 2, 4$, and 8 keV . A stronger thermal pressure support at higher T reduces the deceleration rate and lowers the density gradient, producing a more extended braking region.

Figures 1 and 2 show the velocity $\beta(x)$ and density $\rho(x)$ profiles for a set of constant-temperature atmospheres with $T = 1$ – 8 keV . These models use a fully ionized carbon composition with $(A, Z) = (12, 6)$ and $\mu = 12/7$. The stopping length is found to depend sensitively on the thermal state of the atmosphere. At low temperature ($T = 1$ – 2 keV), the Coulomb drag is low due to the small electron thermal velocity and hence small Coulomb Logarithm, and the inflowing protons decelerate slowly, producing a broad braking zone of thickness $\Delta x \sim$ a few millimeters. As the temperature increases, the electron thermal speed grows, and the Coulomb logarithm increases moderately, leading to a more effective drag force. This behavior results in a progressively more compact braking region and a steeper velocity gradient. The atmospheric response follows the same trend. At higher temperatures, the compression is steeper, and the peak density attained before complete stopping is correspondingly larger. These temperature-dependent density gradients are essential for determining the location of the vacuum resonance and the degree of adiabatic mode conversion, as discussed in the Conclusion.

Physically, increasing the temperature increases the Debye length, which in turn increases the Coulomb Logarithm. This causes the Coulomb drag to be more effective, resulting in the flow losing momentum faster and hence stopping over a shorter distance.

Chemical Composition Influencing Plasma Brake:

To quantify the dependence on ion mass and charge, we compare in Figure 3 hydrogen and carbon atmospheres at a fixed temperature $T = 2 \text{ keV}$. These compositions correspond to $(A, Z) = (1, 1)$ with $\mu = 1/2$ and $(A, Z) = (12, 6)$ with $\mu = 12/7$, respectively. The braking behavior differs strongly between the two cases. In a hydrogen atmosphere, the Coulomb drag is intrinsically weaker because the collision term scales as Z^2 and because the larger number of particles per unit mass (smaller μ) reduces the atmospheric inertia. Consequently, the protons penetrate deeper before being stopped, and the resulting density enhancement is relatively modest. The density gradient is shallow, and the velocity decreases gradually over a distance of order $\sim 1 \text{ cm}$. In contrast, a carbon atmosphere provides both a higher collision rate ($Z^2 = 36$) and a larger mean par-

ticle mass ($\mu = 12/7$). The combination of stronger Coulomb coupling and a more inert medium causes rapid deceleration within ~ 0.1 cm. The atmospheric density correspondingly increases over a smaller spatial scale, producing a much steeper density gradient in the braking layer. This behavior reflects the combined effects of stronger Coulomb coupling and increased atmospheric inertia in high- Z compositions: a larger charge enhances the collision rate, while a higher mean molecular weight reduces the ability of the atmosphere to redistribute the deposited momentum, leading to rapid deceleration and strong local compression.

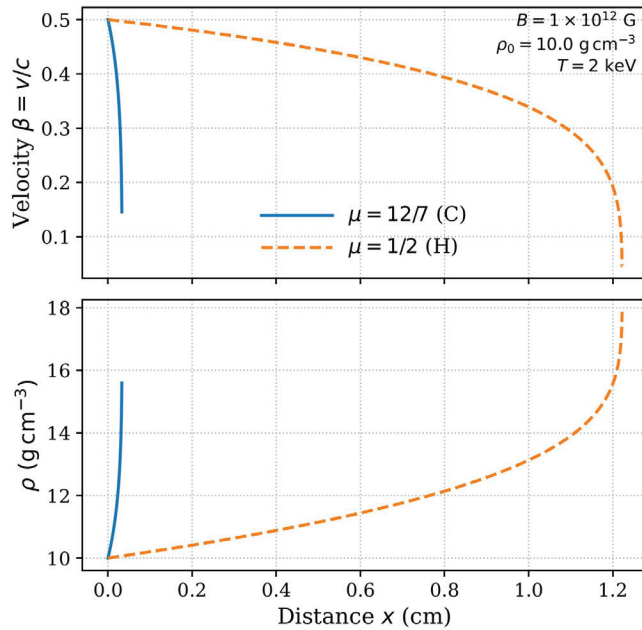


Figure 3: Comparison of velocity $\beta(x)$ and density $\rho(x)$ profiles for two fully ionized compositions—hydrogen ($A = 1$, $Z = 1$, $\mu = 1/2$) and carbon ($A = 12$, $Z = 6$, $\mu = 12/7$)—at a fixed temperature $T = 2$ keV. The differing mean molecular weights and charge states modify both the Coulomb drag (through Z^2) and the hydrostatic compression term (through μ), leading to distinct braking lengths and density structures. All other parameters are the same as in Figures 1 and 2.

Stopping Lengths and Energy Dissipation:

The total stopping length x_{stop} ranges from ~ 0.01 cm in cool, high- Z atmospheres to $\gtrsim 1$ cm in light, hot compositions. In all cases, most of the kinetic energy is dissipated in the bottom half of the braking layer, where both the density and the Coulomb coupling are largest. The fraction of the incident kinetic energy converted into local heating within each depth interval can be computed directly from the drag force and is naturally stored as part of the numerical diagnostics, although we defer a detailed analysis of this energy deposition profile to future work. Overall, the numerical results demonstrate that both temperature and composition play decisive roles in shaping the vertical structure of the atmosphere in sub-critical XRPCs. The steepening or smoothing of the density gradient has a direct influence on radiative transfer, vacuum resonance crossing, and ultimately the polarization degree of the emergent X-ray radiation.

Atmospheric Compression by Coulomb Braking:

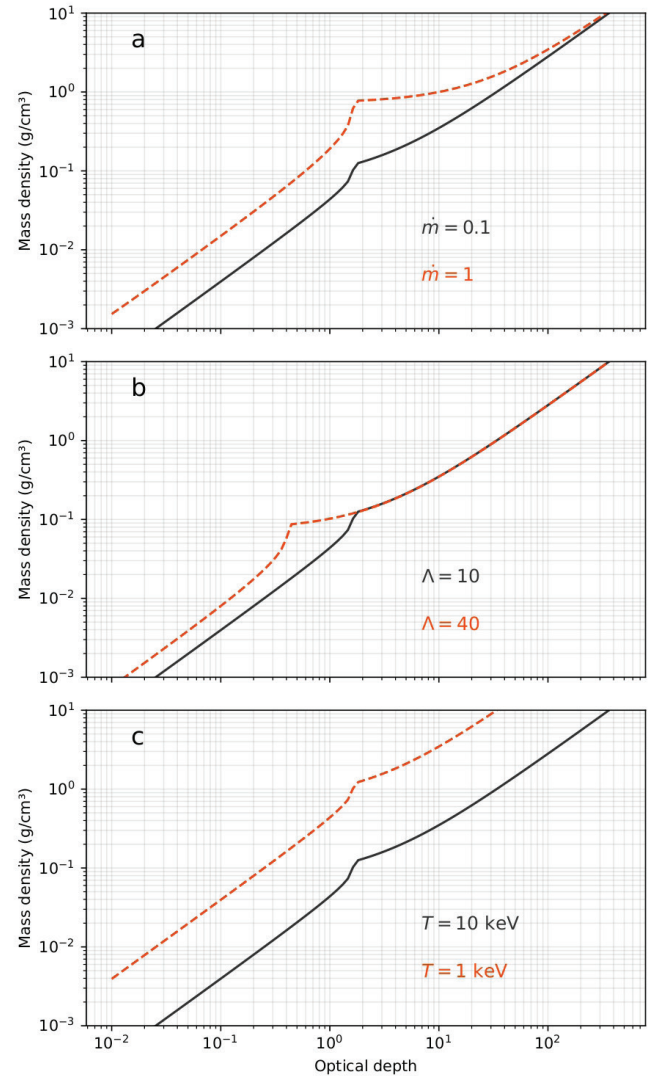


Figure 4: Mass density profiles $\rho(\tau)$ as a function of optical depth for a set of atmospheric models illustrating the influence of (a) the mass accretion rate per unit area, (b) the Coulomb logarithm, and (c) the atmospheric temperature. In all panels, the horizontal axis shows the optical depth with respect to non-magnetic Thomson scattering, while the vertical axis shows the local mass density.

Figure 4 illustrates how Coulomb braking of the accretion flow imprints a characteristic vertical structure on the atmosphere of a strongly magnetized NS. The horizontal axis in all panels shows the optical depth with respect to non-magnetic Thomson scattering, while the vertical axis gives the local mass density. In each model, the inflowing plasma deposits its momentum within a finite stopping layer, causing a distinct compression feature: the density rises steeply once the protons begin to decelerate efficiently, producing a pronounced “knee” in the $\rho(\tau)$ profile. The location and sharpness of this feature directly reflect the depth at which the flow comes to rest. Panel (a) shows the dependence of the atmospheric response on the mass accretion rate per unit area. The black curve corresponds to a ten times lower inflow rate than the red curve, and the resulting compression is accordingly weaker. A larger accretion flux transfers more momentum to the atmosphere

and forces the braking region to greater density and optical depth, yielding a systematically higher density throughout the upper layers. This behavior emphasizes that even sub-critical accretion can substantially reshape the stratification of the atmosphere when the inflow is sufficiently intense. Panel (b) shows how the result changes with the value of the Coulomb logarithm, which quantifies the efficiency of collisional momentum transfer. Increasing $\ln\Lambda$ —from 10 in the black curve to 40 in the red curve—strengthens the deceleration of the flow. As a consequence, the inflowing protons are stopped at smaller optical depths, producing a sharper and more elevated density enhancement in the upper atmosphere. At larger depths, however, both models converge to the same hydrostatic gradient, indicating that the braking mainly alters the structure above the stopping layer while leaving the deep atmosphere essentially unaffected. Panel (c) explores the influence of the atmospheric temperature. A cooler atmosphere (red curve, $T = 1$ keV) has a higher density at a given optical depth due to reduced thermal pressure support, and therefore experiences more efficient Coulomb braking. Although both the hot and cool models exhibit the same qualitative signature of a density jump associated with the termination of the inflow, the entire density profile is shifted upward in the colder case. This systematic offset reflects the competing roles of thermal support and momentum deposition in determining the vertical structure. Overall, Figure 4 highlights that the interplay between accretion dynamics and atmospheric microphysics naturally generates strong compression of the upper layers of the neutron-star atmosphere. The depth and sharpness of the braking-induced density gradient depend sensitively on the accretion rate, the Coulomb coupling strength, and the thermal state of the plasma. These variations modify the local optical depth profile and are therefore expected to influence radiative transfer, vacuum-resonance behavior, and ultimately the observed polarization properties of sub-critical X-ray pulsars.

■ Conclusion

In this work, we have investigated the deceleration of accretion flows in sub-critical X-ray pulsars, focusing on the regime in which radiation pressure is negligible and Coulomb collisions in a strongly magnetized neutron star atmosphere dominate the braking process. Using a self-consistent numerical integration of the coupled velocity and density equations, we have quantified how the accretion flow – atmosphere interaction shapes the vertical structure of the accretion flow base. Our results demonstrate that plasma braking naturally produces a strongly compressed upper atmosphere, with a characteristic steepening of the density gradient once the infalling protons begin to lose momentum efficiently. This compression feature provides a clear dynamical signature of the transition between the decelerating flow and the underlying hydrostatic atmosphere. The location, amplitude, and sharpness of this compression feature depend sensitively on the local plasma parameters. Higher mass accretion rates lead to more substantial atmospheric compression and shift the braking region to larger optical depths (see Figure 4a). Variations in the Coulomb log-

arithm alter the efficiency of collisional momentum transfer, modifying the stopping depth and the structure of the upper layers (see Figure 4b). The thermal state of the atmosphere likewise plays a key role (see Figure 4c): cooler plasmas brake the inflow more rapidly and yield higher peak densities, while hotter atmospheres produce more extended and gradual deceleration zones. These findings have important implications for radiative transfer of polarized X-rays in sub-critical XRPs. The density gradients produced by Coulomb braking regulate the local optical depth and determine whether the normal modes of radiation propagate adiabatically or undergo partial mode conversion near the vacuum resonance.² In particular, because the adiabaticity of mode propagation depends sensitively on the density scale height, the braking-induced modification of $\rho(z)$ can change the probability of non-adiabatic switching between the O- and X-modes near the vacuum resonance, thereby affecting the polarization state of the emerging radiation. The braking-induced stratification, therefore, represents a key physical ingredient in modeling the emergent spectra and polarization signatures of accreting neutron stars. The framework developed here provides a physically consistent description of the braking layer and its dependence on accretion rate, composition, and temperature. The resulting density profiles can be used directly as input for spectral, polarimetric, and atmosphere-modeling calculations. We emphasize that the main contribution of this work is to provide self-consistent vertical structure profiles in the plasma braking regime, which can be straightforwardly coupled to existing radiative-transfer and polarization pipelines. Future work will extend the present approach by incorporating nonconstant temperature profiles, radiative feedback, and multidimensional effects, enabling a more complete description of the accretion footprint in both sub-critical and transitional luminosity regimes.

Future Work:

Several extensions of this work could provide a more complete picture of sub-critical accretion.

First, incorporating self-consistent temperature profiles, calculated from energy deposition and radiative cooling, would replace the isothermal approximation. This would capture the thermal feedback between deceleration and atmospheric structure, which was assumed to be constant in this research.

Second, including radiative feedback, especially through cyclotron cooling and free-free emission, would allow us to model the shift from sub-critical to near-critical states. In these states, radiation pressure starts to affect the dynamics.

Third, extending the model to multiple spatial dimensions, either 2D or 3D, would help us study the lateral spread of the accretion flow. It would also allow us to examine the formation of uneven density structures within the hot spot and how magnetic field geometry influences vertical layering.

■ Acknowledgments

I want to express my deepest gratitude to my mentor, Dr. Alexander Mushtukov, for their invaluable guidance, encouragement, and support throughout this research project. Their

insightful feedback and expertise were instrumental in shaping the methodology and the final research paper.

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