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# Leontief Input-Output Analysis of Agricultural Carbon Emissions and Output in Canada

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ABSTRACT: Social production and the environment are fundamental elements that humans depend on to live. They're both opposite and connected. Figuring out how to balance them has always been a lasting problem for people. In this research, we employed the standard Leontief model to examine the relationship between agricultural production and carbon emissions in Canada. With this model, we conducted a set of sensitivity studies, testing various strategies to reduce emissions. Ultimately, we developed some suggestions for the future development of agriculture.

KEYWORDS: Mathematics, Linear Algebra, Input-Output Analysis, Agricultural Carbon Emissions, Sectoral Environmental Impact.

## Introduction

Today, with the advanced development of social technology, environmental issues have become increasingly important. For example, the greenhouse effect caused by carbon emissions has begun to endanger people's survival. Agricultural production is essential for human sustenance. However, it is also a major contributor to carbon emissions. According to a 2021 report by the Food and Agriculture Organization of the United Nations (FAO), agricultural activities account for approximately 30% of global anthropogenic greenhouse gas emissions.[1] Hence, one can see the significant role agriculture plays in the global carbon footprint. Therefore, how to best handle the contradiction between environmental protection and agricultural production has become an important issue on our agenda that needs to be addressed urgently.

There are many types of agricultural activities, mainly involving crop cultivation, animal husbandry, forestry, and fisheries.[2,3] Among these, livestock production has the largest share of carbon emissions. Therefore, it is necessary to study the specific details of agricultural production. We need to refine and optimize the types of production, adjust the internal sector planning of agriculture, and quantify the environmental impact of each sector's production. By doing so, we can minimize the environmental impact of agricultural production while continuously promoting its healthy development to ensure an adequate food supply.

To understand the internal relationships among agricultural activities and their associated environmental impacts, we can employ the Leontief model (also known as input-output analysis).[4] This model can help analyze the flow of resources within each sector of agricultural production, predict total output based on total demand, and evaluate the input allocation among internal sectors in response to changes in demand for each production department. Most importantly, this model can be further extended to connect the total agricultural output with the environmental impact intensity through simple vector coefficients,[5] integrating all aspects of agricultural production, output, and environmental intensity. This method, hence, provides a solid foundation for optimizing subsequent agricultural policies.

The manuscript is structured as follows: The Methods section describes the establishment of the input-output table, including the determination of data and relevant methodologies. The Results and Discussion section presents the internal relationships within the agricultural sector and its external environmental impacts. These findings are then utilized in sensitivity analyses to obtain optimized configurations for agricultural activities, aiming to minimize environmental footprints. The conclusions are synthesized in the final section, summarizing key insights and implications.

## Methods

The main part of conducting Input-Output analysis is to construct an input-output table for designated sectors, which includes the final demand of each sector and its associated environmental pollution quantified in cost terms. With this data, a technical coefficient matrix can be derived to mathematically link the final demand to each sector's total output via linear algebraic methods. This framework further enables sensitivity analyses to evaluate the impact of adjustments in agricultural activities on environmental pollution.

### *Core Data Table:*

Considering the data availability and integrity, we refer to the annual reports on gross domestic product by industry and human activities and environment, which were released in 2012 and 2014, respectively, and can be openly accessed on Statistics Canada.[5,6] According to the report, agricultural activities primarily encompass three sectors: crop production, animal production, and support for fishing and forestry. Here, we refer to "Animal production" as animal husbandry, including the breeding, raising, and management of livestock. Additionally,

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we combine fishing and forestry production (FFS) because these are minor sectors.

Table 1 shows total output and final demand, represented by X and Y, respectively. For the specific total output in the table, we use the average gross product in 2012, as the data in the gross product report have been seasonally adjusted to annual rates. Notably, the final demand is temporarily assumed to be 60% of the associated output. This assumption is made because this data will later be updated to ensure consistency among the output, inputs, and final demand, due to the incomplete input flows in the input-output table, as shown in Table 2.

**Table 1:** Statistics of total output, final demand (unit: millions of chained 2002 dollars). The role of basic data on total output and final demand. The total output and final demand values in this table allow us to establish an input-output relationship between the sectors. It also shows the relative levels of output between the sectors, and which contribute most economically.

| Sector | Total Output (X) | Final Demand (Y) |
|---|---|---|
| Crop Production | 18,112 | 10,868 |
| Animal Production | 5,084 | 3,050 |
| Fishing/Forestry Support | 2,667 | 1,600 |

***Input-Output Table and Technical Coefficients:***

The Leontief model is based on an Input-Output table, which describes the flow of agricultural commodities and production factors among all individual sectors. The model's structure is presented in Table 2. Columns (or column vectors) represent the inputs that a sector receives from other sectors (including itself). Rows represent the allocation of a sector's total output across agricultural activities. In the table, the intermediate consumption term $Z_{ij}$ represents the monetary value of agricultural commodities and production factors that sector *j* directly consumes from s sector *i*.. Eq. (1) defines the consumption ratio $A_{ij}$

$$A_{ij} = \frac{Z_{ij}}{X_j}, \tag{1}$$

where $X_j$ is the total output of sector j shown in Table 2. This means that for every unit of output produced by sector *j*, it directly requires 15% of a unit worth of inputs from sector *i*. Since we do not know the specific consumption terms $Z_{ij}$ for 2012 or other years, we instead refer to the 2019 symmetric supply annual report[2] to obtain all the self-consumption ratios $A_{ii}$ and the cross-sector ratio $A_{12}$, which are highlighted in bold in the table. The other cross-sector ratios, $A_{ij}$, are derived from reasonable assumptions based on general patterns of agricultural activities. With these ratios, $A_{ij}$, one can, in turn, determine the associated terms, $Z_{ij}$. Additionally, in Eq. (2), the relation between the total output X and the final demand Y is given by

$$Y_j = X_j - \sum_{i=1}^{n} Z_{ij}, \tag{2}$$

where n represents the number of individual sectors. Although Table 1 already displays the final demand term, this term must be updated using Eq. (3), as shown in Table 2.

**Table 2:** The definition of input-output relationships and technical coefficients (unit: million dollars). Here, the input-output relationships are established and define the demand values. These relationships define how much each sector influences the others.

| into<br>from | Sector *j*=1:<br>Crop Production | Sector *j*=2:<br>Animal Production | Sector *j*=3:<br>Fishing/Forestry | Total Output (X) | Demand (Y) |
|---|---|---|---|---|---|
| Sector *i*=1:<br>Crop Production | $Z_{11}$ = 5,071<br>($\mathbf{A_{11} = 0.28}$) | $Z_{12}$ = 1,627<br>($\mathbf{A_{12} = 0.32}$) | $Z_{13}$ = 747<br>($A_{13}$ = 0.28) | 18,112 | 10,868 |
| Sector *i*=2:<br>Animal Production | $Z_{21}$ = 1,449<br>($A_{21}$ = 0.08) | $Z_{22}$ = 1,627<br>($\mathbf{A_{22} = 0.32}$) | $Z_{23}$ = 213<br>($A_{23}$ = 0.08) | 5,084 | 1627<br>(3,050) |
| Sector *i*=3:<br>Fishing/Forestry | $Z_{31}$ = 724<br>($A_{31}$ = 0.04) | $Z_{32}$ = 203<br>($A_{32}$ = 0.04) | $Z_{33}$ = 987<br>($\mathbf{A_{33} = 0.37}$) | 2,667 | 720<br>(1,600) |
| Total Input | 7,244 | 3,457 | 1,947 | —— | —— |

Note: The coefficients $A_{ij}$ are unitless. Y is updated; the term in parentheses is the previous value.

Finally, in Eq. (3), we have the Leontief matrix formation as given by

$$\mathbf{X} = \mathbf{AX} + \mathbf{Y}, \tag{3}$$

and reorganizing this equation yields

$$\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{Y}, \tag{4}$$

where A and I are 3x3 matrices, and the diagonal entries in **I** are 1. **X** and **Y** are column vectors.

The term $(\mathbf{I} - \mathbf{A})^{-1}$ is also called the Leontief inverse matrix. Using Eq. (4), the total output can be determined for a given final demand while keeping the consumption matrix **A** constant.

***$CO_2$ Emissions and Farm Output:***

Greenhouse gas emissions from agricultural activities are typically quantified in terms of carbon dioxide equivalent ($CO_2e$). The amount of $CO_2e$ varies across different farming sectors, encompassing both direct and indirect emissions. For example, livestock production generates the highest $CO_2e$ emissions compared to crop production and forestry/fishery support activities. These emissions primarily consist of direct emissions (e.g., enteric fermentation and manure management) and indirect emissions (e.g., feed processing and related supply chain activities). In crop production, indirect emissions (such as those from agricultural machinery, fuel, and fertilizer production) often exceed direct emissions from agricultural soils. Based on Table 1 and the reported statistics of emissions from human activities,[5] a summary table of agricultural emissions and output is presented in Table 3.

**Table 3:** Data about $CO_2$ emissions and farm output. The emissions shown help decide and determine which agricultural sectors would be most efficient to focus on in terms of emission reduction.

| Sector | Total Output (X) (millions of dollars) | Emissions ($MtCO_2e$) (Direct + Indirect) | Emission ratio (e) ($tCO_2e$/$1K) |
|---|---|---|---|
| Crop Production | 18,112<br>(70%) | 17.3→30.9%<br>(**6.4**+10.9) | 0.955 |
| Animal Production | 5,084<br>(19.7%) | 37.1→66.3%<br>(**34** + 3.1) | 6.39 |
| Fishing/Forestry Support | 2,667<br>(10.3%) | 1.6→2.8%<br>(**0** + 1.6) | 0.6 |
| Total | 25,863 | 56 (40.4 + 15.6) | —— |

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According to Table 3, both total and direct emissions are reported (shown in bold). In contrast, indirect emissions are assumed to be allocated based on their associated percentage of total output. We also include the rate of carbon emissions in the Emissions column. To quantify the relationship between output and greenhouse gas emissions, we use the ratio of emissions to output, denoted by **e**. In terms of linear algebra and with reference to Eq. (4), the total emissions can be expressed as

$$\mathbf{TE} = \mathbf{eX} = \mathbf{e}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{Y}\,, \tag{5}$$

Where **e** is a 1×3 row vector. Considering the final demand vector **Y** in Eq. (5), we use the equation

$$\mathbf{f} = \mathbf{e}(\mathbf{I} - \mathbf{A})^{-1}, \tag{6}$$

to measure each agricultural sector's emission intensity **f**, where **f** is a row vector. With the emission intensity **f** held constant for a given final demand **Y**, the total $CO_2$ emissions are calculated as

$$\mathbf{TE} = \mathbf{fY}\,. \tag{7}$$

Again, it should be noted that the vector **e** represents the emission intensity of output (emissions per unit output), while the vector **f** maps final demand to total emissions.

## Results and Discussion

The purpose of this research is to clarify relationships between internal agricultural activities and their gas emissions, proposing emission-reduction adjustments that maintain core outputs. From Table 3 above, it can be observed that animal production generates substantial emissions, while its output accounts for only ~20% of total agricultural output. Therefore, priority should be given to adjustments targeting animal production, alongside complementary measures for other sectors. Through a sensitivity analysis, we investigate changes in emissions and output resulting from adjustments to specific improvement measures.

***Case 1: Reduction in Livestock Demand:***

Firstly, we reduce the final demand of the animal sector to decrease emissions directly. For example, we assume the final demand from the animal sector is reduced by 20%, such that the column vector **Y** in the table becomes $\boldsymbol{Y}_{\mathbf{new\text{-}1}}$=[10868, 1301.6, 720]$^T$. Given the established consumption coefficient matrix **A**, the associated output vector $\boldsymbol{X}_{\mathbf{new\text{-}1}}$ can be calculated using Eq. (4) as

$$\mathbf{X}_{new-1} = \left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.28 & 0.32 & 0.28 \\ 0.08 & 0.32 & 0.08 \\ 0.04 & 0.04 & 0.37 \end{bmatrix} \right)^{-1} \begin{bmatrix} 10868 \\ 1301.6 \\ 720 \end{bmatrix} = \begin{bmatrix} 18017.6 \\ 4335.3 \\ 2562.1 \end{bmatrix}. \tag{8}$$

Given Eqs. (5–7), the adjusted demand $\boldsymbol{Y}_{\mathbf{new\text{-}1}}$ directly yields the total emissions

$$\mathbf{TE}_1 = \mathbf{eX}_{new-1} = \begin{bmatrix} 0.955 & 6.39 & 0.6 \end{bmatrix} \begin{bmatrix} 18017.6 \\ 4335.3 \\ 2562.1 \end{bmatrix} = 46.45\,(\mathrm{MtCO_2e}). \tag{9}$$

As seen from Eq. (8) and Eq. (9), in this case, with a 20% reduction in demand for animal production, the decrease in total emissions is 56–46.45=9.55 (MtCO$_2$e). Simultaneously, it can be observed that a change in the animal sector's final demand alone affects the total output of the other two sectors due to internal dependencies.

***Case 2: Livestock Feed Improvement:***

From Table 3, it is evident that in the animal sector, direct emissions (34 MtCO$_2$e) account for approximately 60% of the total emissions (56 MtCO$_2$e), primarily due to enteric fermentation and manure management. In terms of animal digestion, improving the feed formula and feeding quality can reduce costs and, most importantly, decrease emissions. In this case, we assume that feed improvement results in a 20% decrease in the emission intensity $\mathbf{e}_2$; thus, $\mathbf{e}_2$ becomes 6.39 × (1–0.2) =5.112 tCO$_2$e/$1k (see Table 3). We can then obtain the final total emissions from Eq. (5) with the final output **X** kept constant

$$\mathbf{TE}_2 = \mathbf{e}_{new}\mathbf{X} = \begin{bmatrix} 0.955 & 5.112 & 0.6 \end{bmatrix} \begin{bmatrix} 18112 \\ 5084 \\ 2667 \end{bmatrix} = 44.89\,(\mathrm{MtCO_2e}). \tag{10}$$

This method is also very straightforward for reducing emissions. Additionally, from the Statistics Report,[5] we know that the promotion of no-till technology is also able to reduce the crop emission intensity $\mathbf{e}_1$, automatically leading to a decrease in total emissions seen from Eq. (5). Since the calculation is similar to Eq. (10), we did not add the corresponding process. Generally, no-till technology is often combined with livestock feed improvement. Notably, this feed improvement strategy aligns with the Porter Hypothesis[7,8] in environmental economics, as it achieves emission reduction while optimizing resource use (e.g., reducing feed waste), turning environmental constraints into drivers of production efficiency.

***Case 3: Optimizing Intermediate Feed Consumption in Animal Production:***

Here, we still focus on the animal sector, but with attention to the intermediate consumption coefficient A. Similar to the feeding scenario in Case 2, we know that element $\mathbf{A}_{12}$ (see Table 2) quantifies the amount of output from the crop production sector ($i$ = 1) that is consumed as an intermediate input by the animal production sector ($j$ = 2). Therefore, it is possible that with improved feed use efficiency, the associated consumption is reduced, for example, by 20%. Thus, the original element $\mathbf{A}_{12}$ in the table becomes $\mathbf{A}_{\mathbf{12\_new}}$=0.32×0.8=0.256. From Eq. (4), with demand Y unchanged, the final output is calculated as $\mathbf{X}_{new-2} = (\mathbf{I} - \mathbf{A}_{new})^{-1}\mathbf{Y}$ and

$$\mathbf{X}_{new-2} = \left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.28 & 0.256 & 0.28 \\ 0.08 & 0.32 & 0.08 \\ 0.04 & 0.04 & 0.37 \end{bmatrix} \right)^{-1} \begin{bmatrix} 10868 \\ 1627 \\ 720 \end{bmatrix} = \begin{bmatrix} 17799.76 \\ 4789.9 \\ 2577.1 \end{bmatrix}. \tag{11}$$

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The updated output $X_{new\text{-}2}$ is less than the original values. We then use Eq. (5) to derive the associated total emissions with the original emission intensity e kept unchanged, as given by

$$\mathbf{TE}_3 = \mathbf{eX}_{new-2} = \begin{bmatrix} 0.955 & 6.39 & 0.6 \end{bmatrix} \begin{bmatrix} 17799.76 \\ 4789.9 \\ 2577.1 \end{bmatrix} = 49.15\,(\mathrm{MtCO_2e}). \qquad (12)$$

From this calculation in Eq.(11) and Eq.(12), we can see that optimizing intermediate feed consumption can also contribute to reducing emissions.

***Comparison of Agricultural Carbon Emission Reduction Strategies:***

The outcomes of using different emission reduction strategies are summarized in Table 4. As can be seen, reducing direct emission intensity (case 2) yields the highest emission reduction. However, these strategies cannot be compared on equal terms due to varying implementation challenges and feasibility. For instance, reducing livestock demand (case 1) is straightforward to implement in the short term but may conflict with long-term food security goals. In contrast, the two technology-driven approaches (cases 2 and 3) are more sustainable as they reduce emissions without compromising production scales.

**Table 4:** Summary of carbon emission reduction strategies. The reduction rates lead us to be able to find the most efficient way to reduce emissions. The highest reduction rate indicates the most efficient method.

| Case # | Strategy | Emissions ($\mathrm{MtCO_2e}$) | Reduction rate |
|---|---|---|---|
| 1 | Reduce Livestock demand: $\mathbf{Y}_j$（↓20%） | 46.45 | 17.1% |
| 2 | Decrease emission intensity: $\mathbf{e}_j$ (↓20%) | 44.89 | 19.8% |
| 3 | Optimize consumption coefficient: $\mathbf{A}_{ij}$ (↓20%) | 49.15 | 12.2% |
| Ref. | Baseline | 56 | 0 |

Thus, for long-term agricultural sustainability, greater efforts should be directed toward developing and scaling these technological solutions. Not only do they align with the need to maintain maximum demand for agricultural products, but they also foster efficiency gains that can be integrated into broader farming systems, ensuring both emission reductions and stable productivity.

## Conclusion

In this article, we used the Leontief model to study how agricultural output and carbon emissions are connected, and how different parts of farming (like growing crops and raising animals) affect each other. Although other studies have examined the impact of human activities on the environment using input-output methods, our work employs the standard Leontief Input-Output method. This helps us focus on finding ways to reduce emissions by theoretically changing aspects of the industry. The main conclusions are summarized below:

(1) The Leontief model clearly shows how different parts of farming (like crops and livestock) are linked, and how more output can mean more carbon emissions.

(2) We did several sensitivity studies on cutting emissions, and they showed that different methods have different efficiencies in reducing emissions.

(3) We compared these strategies. Reducing demand for animal products is relatively easy to do, but it may mean consuming less food. The other two methods (better feed and using crops more efficiently) are better for the future because they cut emissions without reducing food output.

## Future Research

Most parts of the intermediate consumption matrix were assumed because there isn't enough available data. Moreover, much of the data used here is approximately 10 years old. With new technologies and changes in society over this time, this old data might not be accurate anymore. Therefore, future studies could conduct a similar analysis using the latest data. That would make the predictions more reliable. It might also be helpful to examine other aspects of agriculture, such as the use of energy on farms or the processing of food after harvesting. Adding cost checks, such as the cost of each emission reduction method, could also make the research more useful for real-world decisions.

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